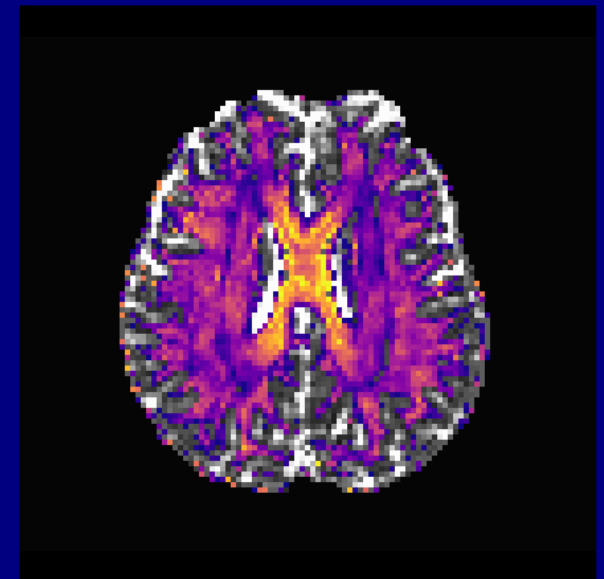


Introduction to: DWI + DTI

AFNI Bootcamp (SSCC, NIMH, NIH)



Outline

- + DWI and DTI
 - Concepts behind diffusion imaging
 - Diffusion imaging basics in brief
 - Connecting DTI parameters and geometry
 - Role of noise+distortion → DTI parameter uncertainty

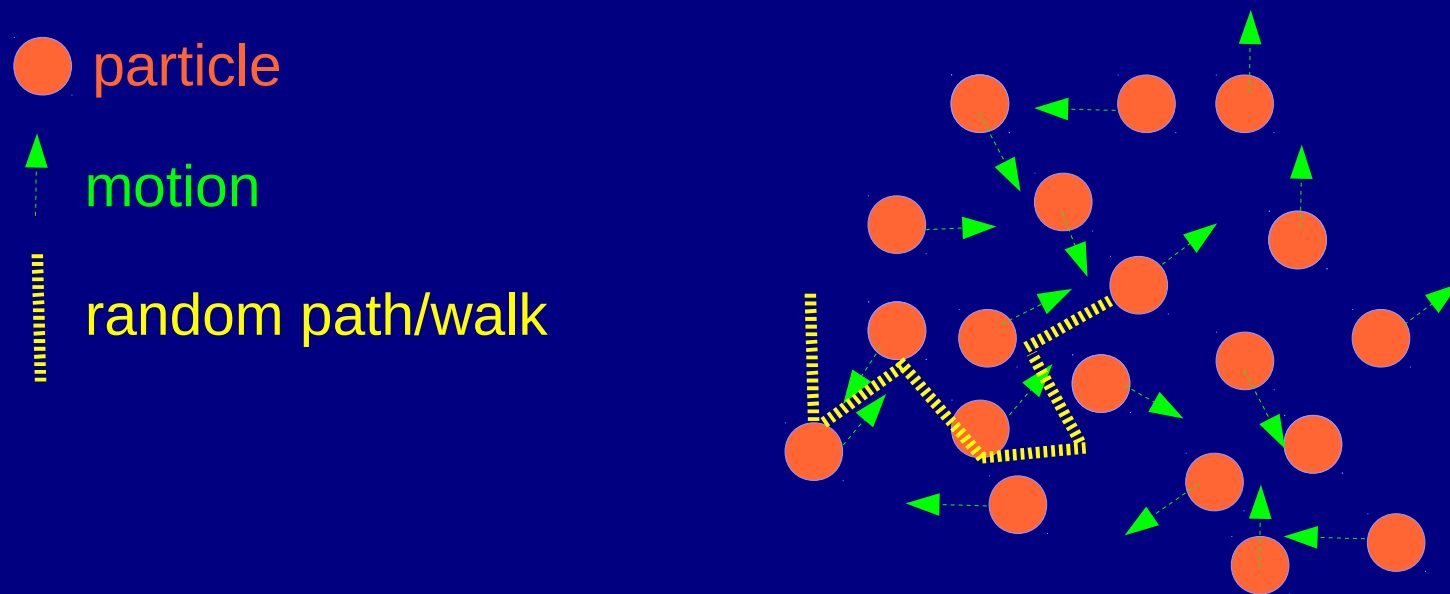
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Tensor: a mathematical object (a matrix) to store information
→ here, quantifying particle spread in all directions

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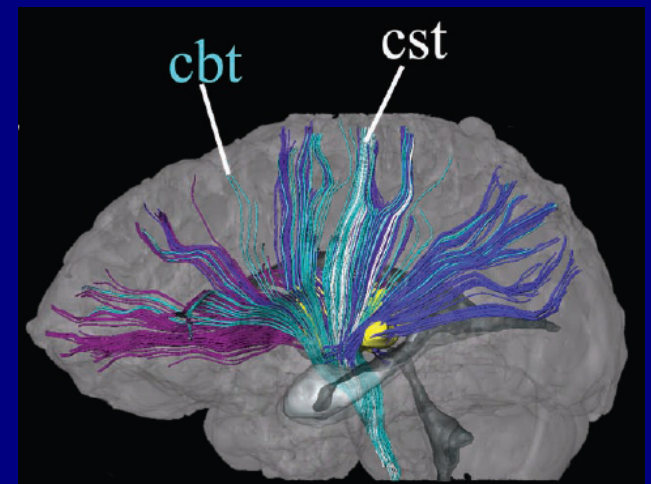
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Tensor: a mathematical object (a matrix) to store information
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Imaging: quantifying brain properties
→ here, esp. for white matter



The DTI model:

Assumptions and relation to WM properties

Diffusion as environmental marker

Diffusion: random (Brownian) motion of particles → mixing or spreading

Ex: unstirred, steeping tea (in a large cup):



Diffusion as environmental marker

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Empty cup, no structure:

Atoms have equal probability of movement any direction

→ spherical spread of concentration

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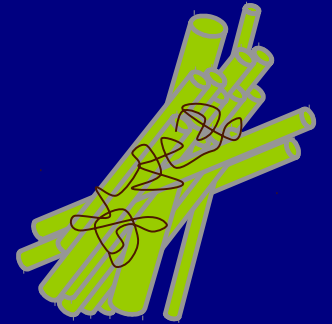
Unequal probabilities of moving in different directions
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→ **Diffusion shape tells of structure presence and spatial orientation**

Local Structure via Diffusion MRI

(In brief)

1) Random motion of molecules affected by local structures

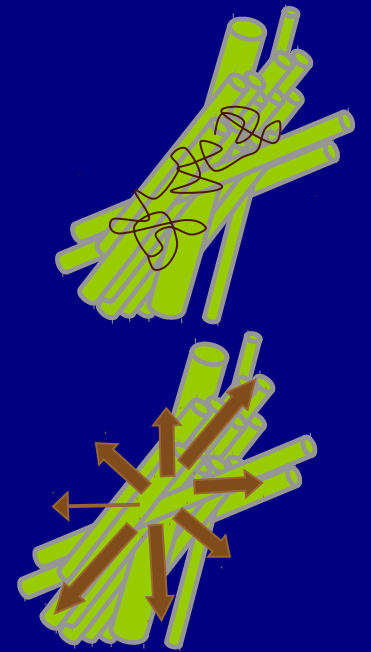


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Local Structure via Diffusion MRI

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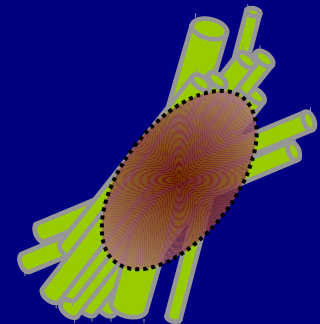
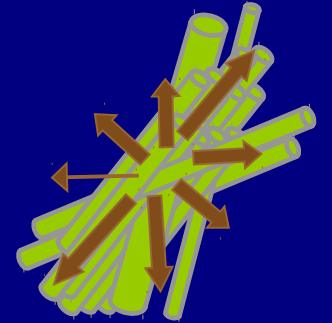
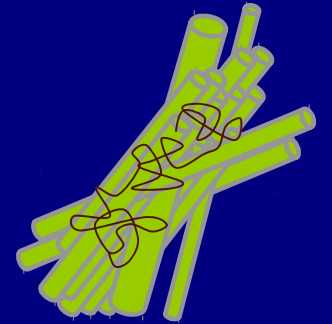
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+ one direction:

DTI (Diffusion Tensor Imaging)



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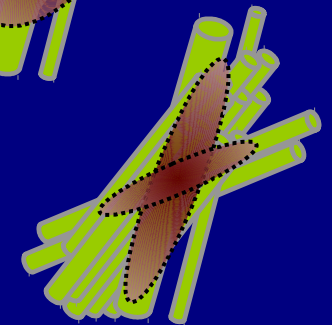
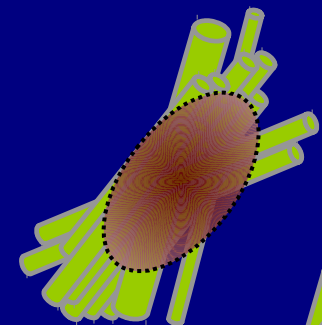
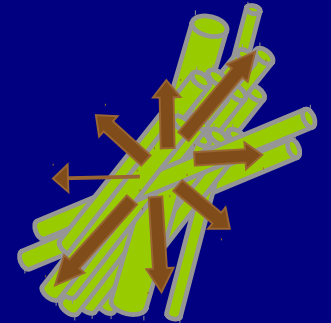
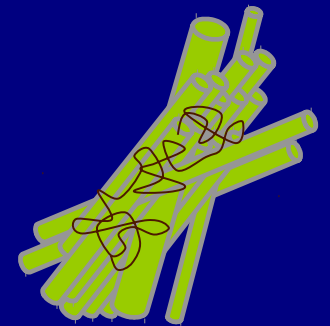
+ one direction:

DTI (Diffusion Tensor Imaging)

+ ≥ 1 direction:

HARDI (High Angular Resolution Diffusion Imaging)

Qball, DSI, ODFs, ball-and-stick, multi-tensor, CSD, ...



Diffusion in MRI

Mathematical properties
of the matrix/tensor:

$$\mathbf{D} = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}$$

Having: 3 eigenvectors: \mathbf{e}_i
3 eigenvalues: λ_i

- Real-valued
- Positive definite ($\mathbf{r}^T \mathbf{D} \mathbf{r} > 0$)
 $\mathbf{D} \mathbf{e}_i = \lambda_i \mathbf{e}_i, \quad \lambda_i > 0$
- Symmetric ($D_{12} = D_{21}$, etc)
6 independent values

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Geometrically, this describes
an ellipsoid surface:

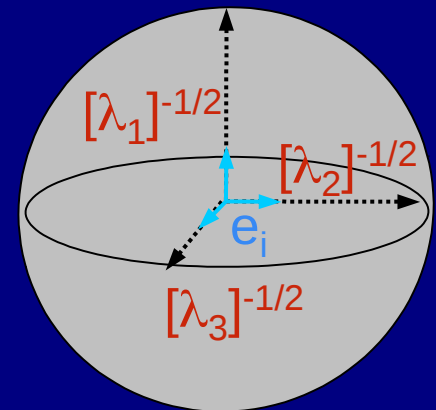
$$C = D_{11}x^2 + D_{22}y^2 + D_{33}z^2 + 2(D_{12}xy + D_{13}xz + D_{23}yz)$$

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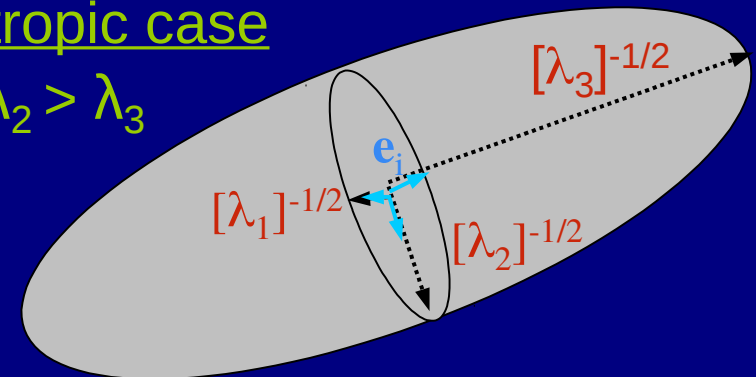
isotropic case

$$\lambda_1 = \lambda_2 = \lambda_3$$



anisotropic case

$$\lambda_1 > \lambda_2 > \lambda_3$$



DTI: ellipsoids

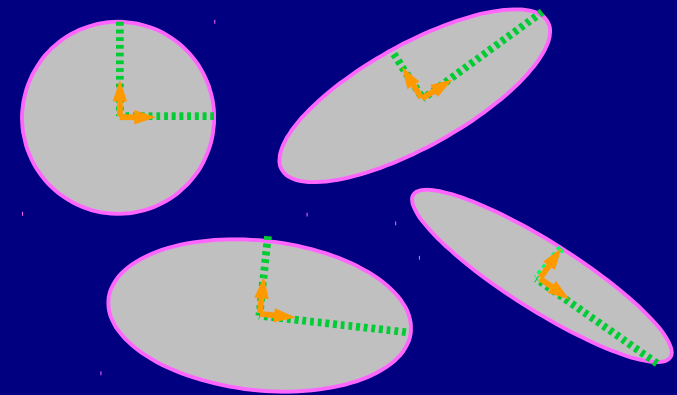
Important mathematical properties of the diffusion tensor:

+ Help to picture diffusion model:

tensor \mathbf{D} → ellipsoid surface

eigenvectors \mathbf{e}_i → orientation in space

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DTI: ellipsoids

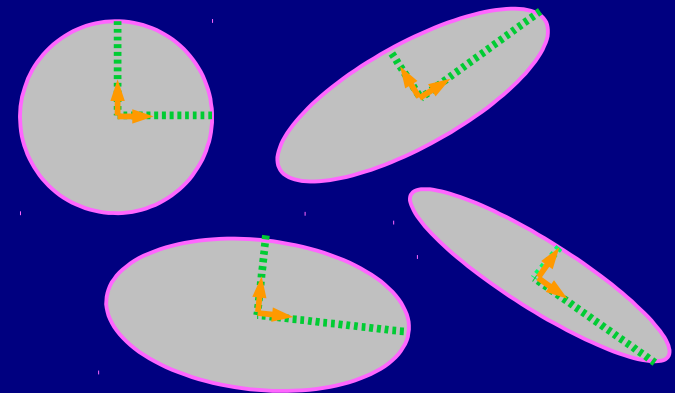
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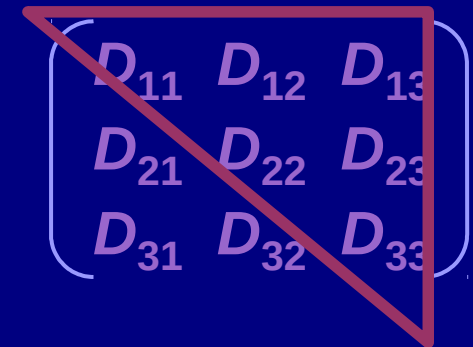
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+ Determine the minimum number of

DWIs measures needed (6 + baseline)



DTI: ellipsoids

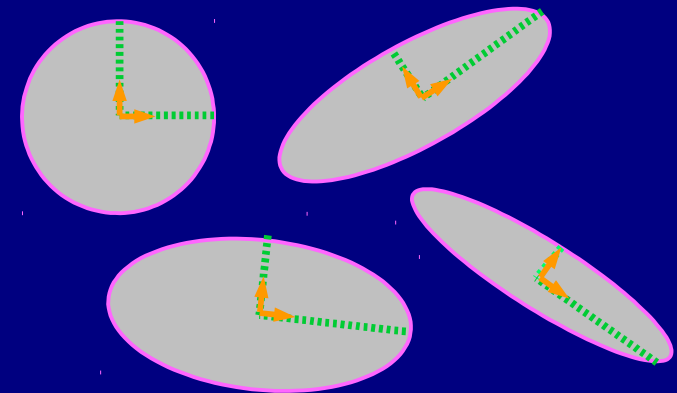
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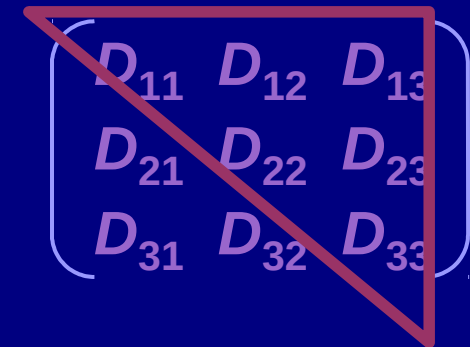
tensor \mathbf{D} \rightarrow ellipsoid surface

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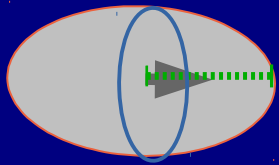
+ Determine much of the processing and noise minimization steps

“Big 5” DTI ellipsoid parameters

Main quantities of diffusion (motion) surface

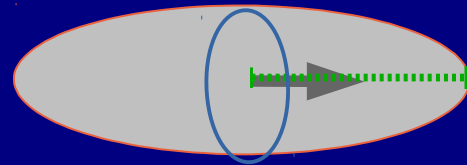
first eigenvalue, L1

(= λ_1 , parallel/axial diffusivity, AD)



$L1_1$

<

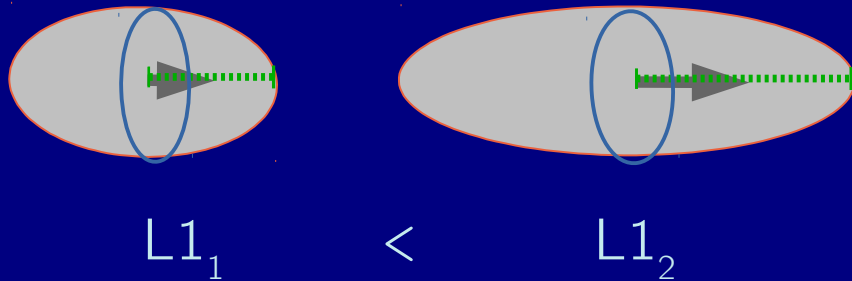


$L1_2$

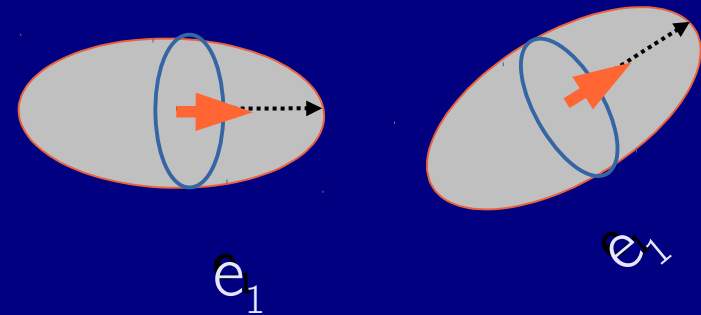
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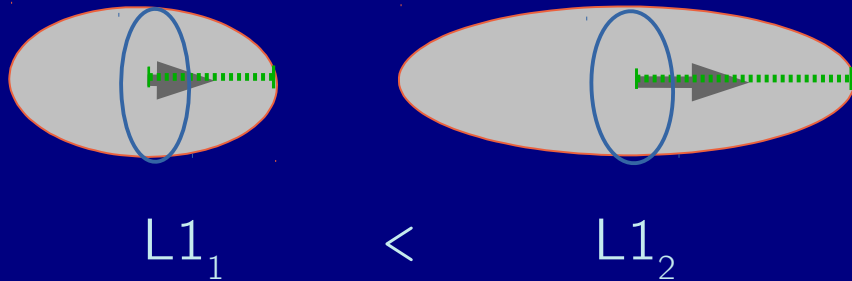
first eigenvector, e_1
(DT orientation in space)



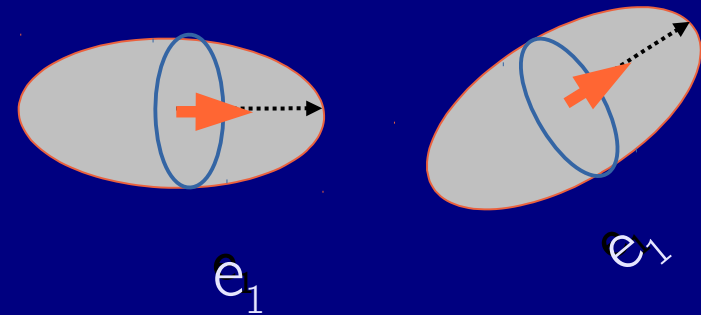
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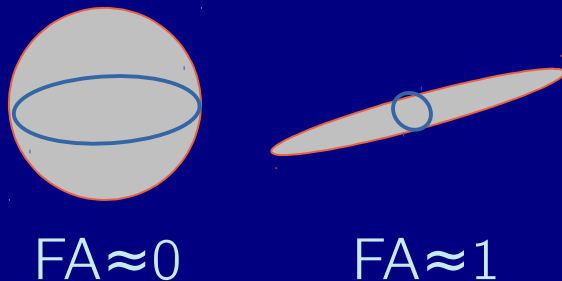
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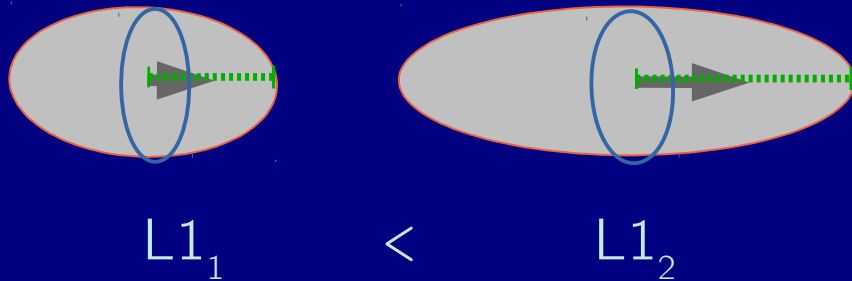
Fractional anisotropy, FA
(stdev of eigenvalues)



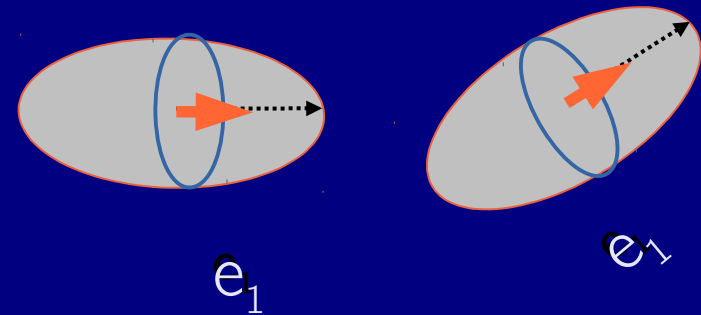
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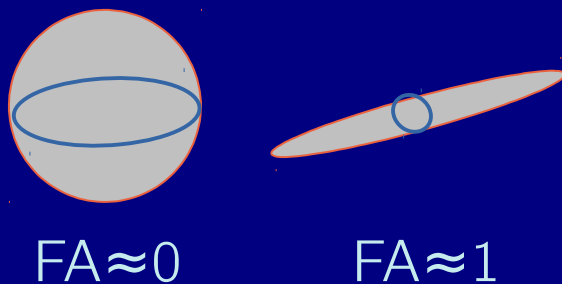
first eigenvalue, $L1$
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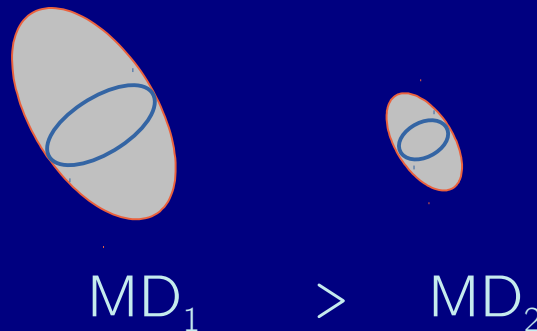
first eigenvector, e_1
(DT orientation in space)



Fractional anisotropy, FA
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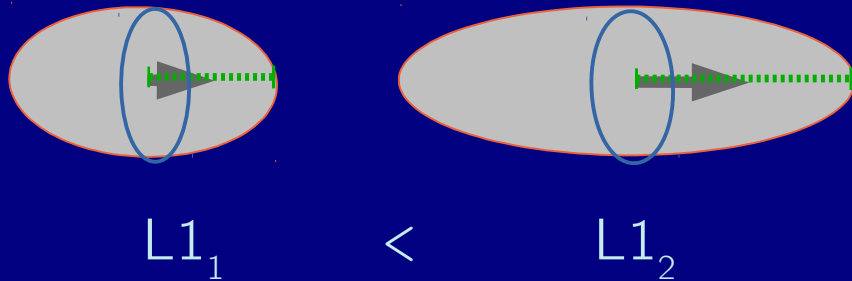
Mean diffusivity, MD
(mean of eigenvalues)



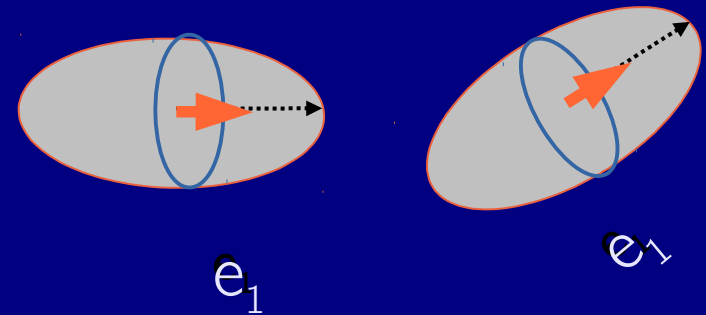
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Main quantities of diffusion (motion) surface

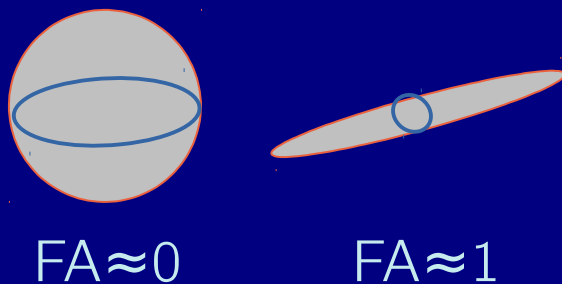
first eigenvalue, L1
(= λ_1 , parallel/axial diffusivity, AD)



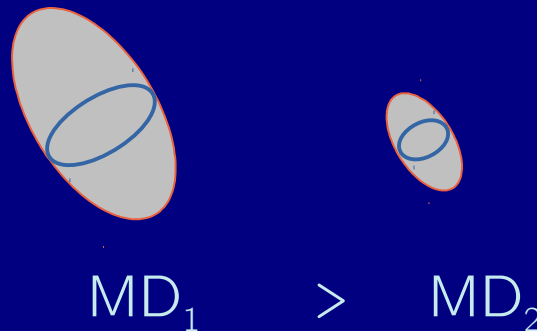
first eigenvector, e₁
(DT orientation in space)



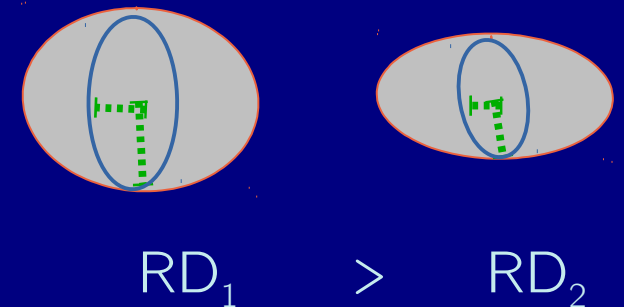
Fractional anisotropy, FA
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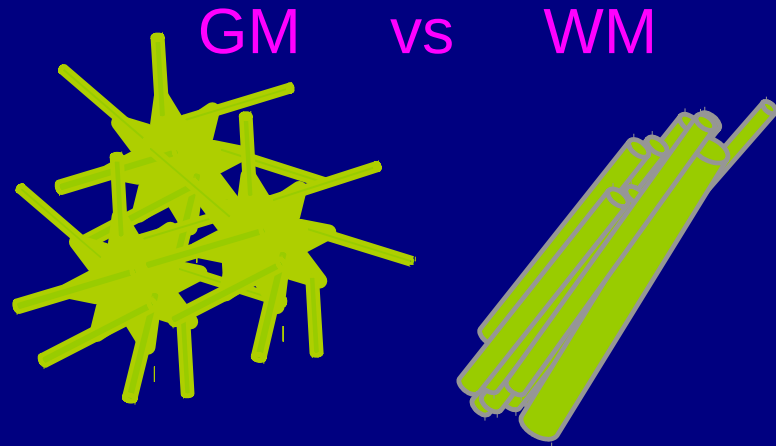
Mean diffusivity, MD
(mean of eigenvalues)



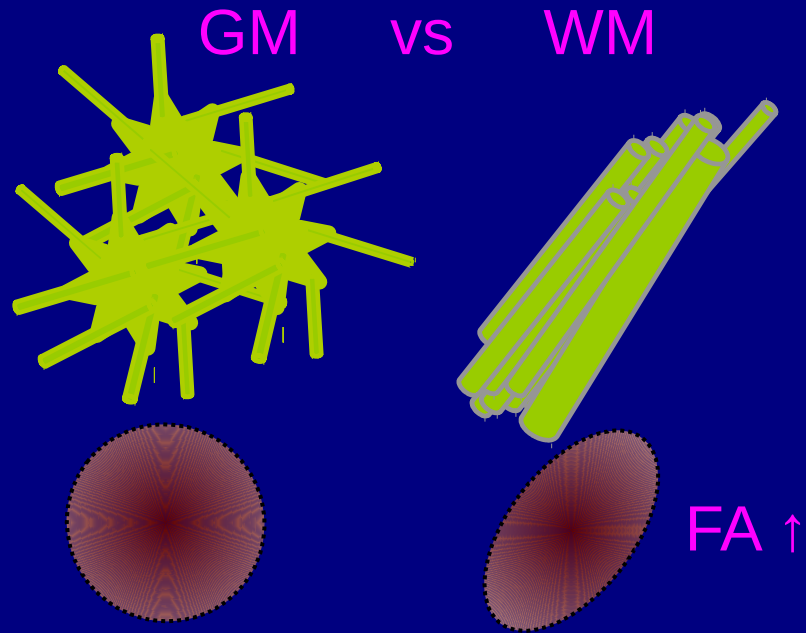
Radial diffusivity, RD
(= $(\lambda_2 + \lambda_3)/2$)



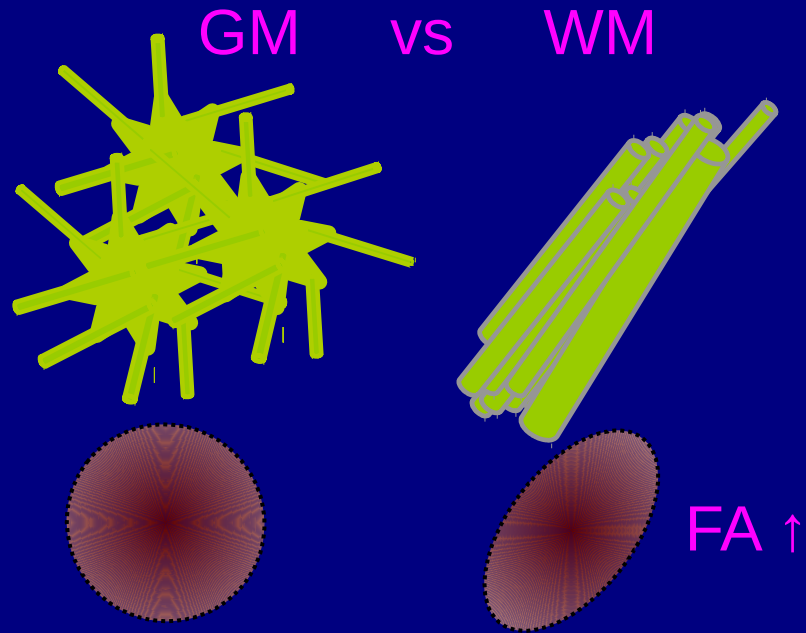
Cartoon examples: white matter \leftrightarrow FA



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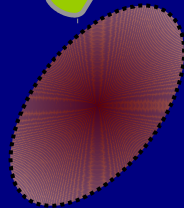
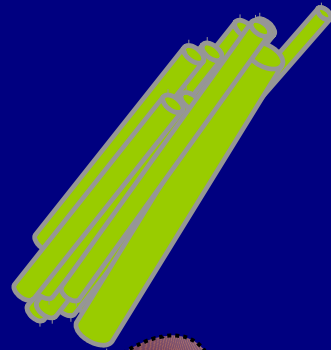
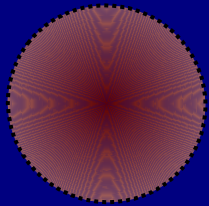
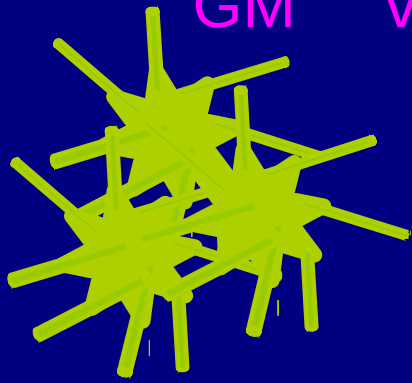


Cartoon examples: white matter \leftrightarrow FA



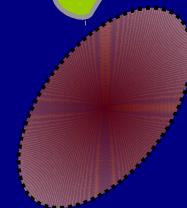
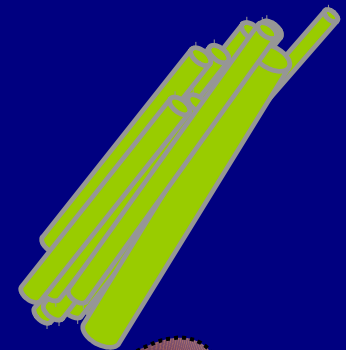
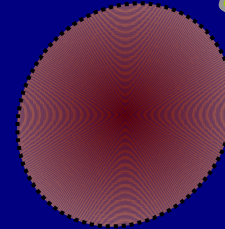
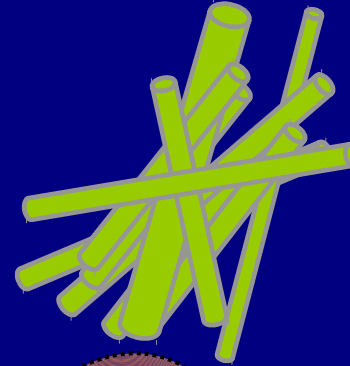
Cartoon examples: white matter \leftrightarrow FA

GM vs WM



FA \uparrow

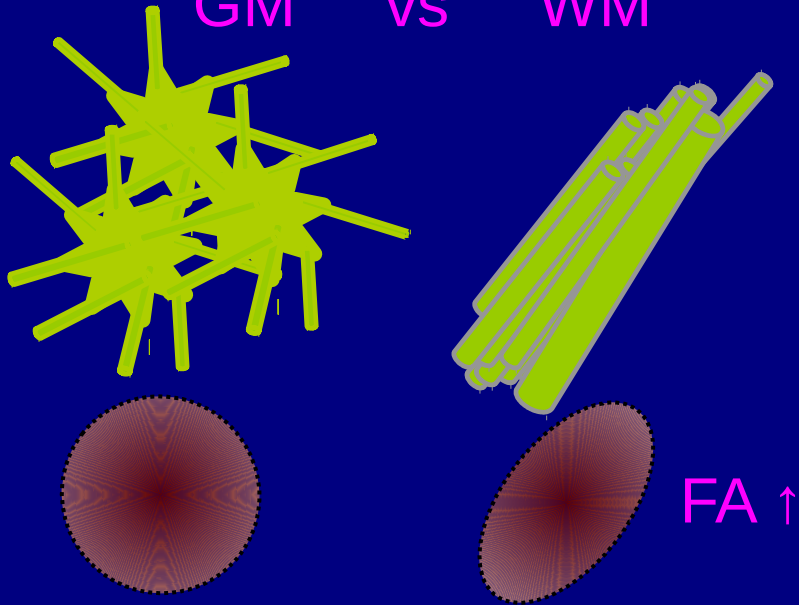
WM bundle organization



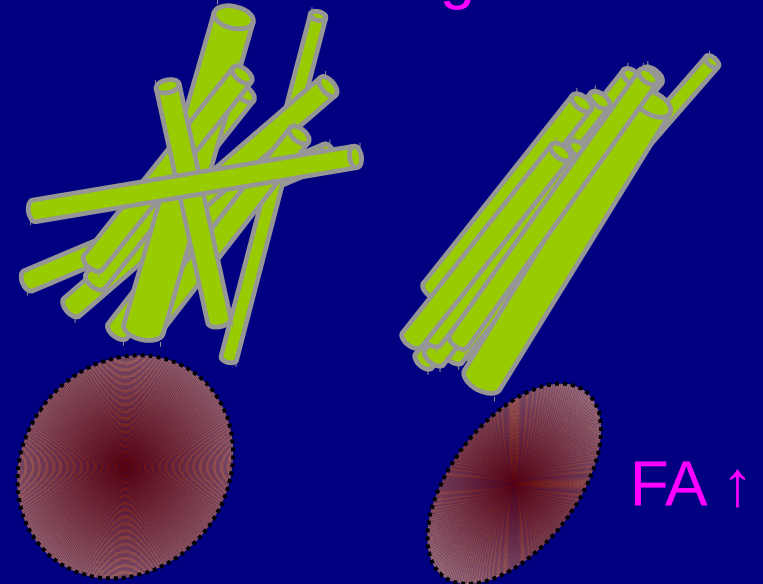
FA \uparrow

Cartoon examples: white matter \leftrightarrow FA

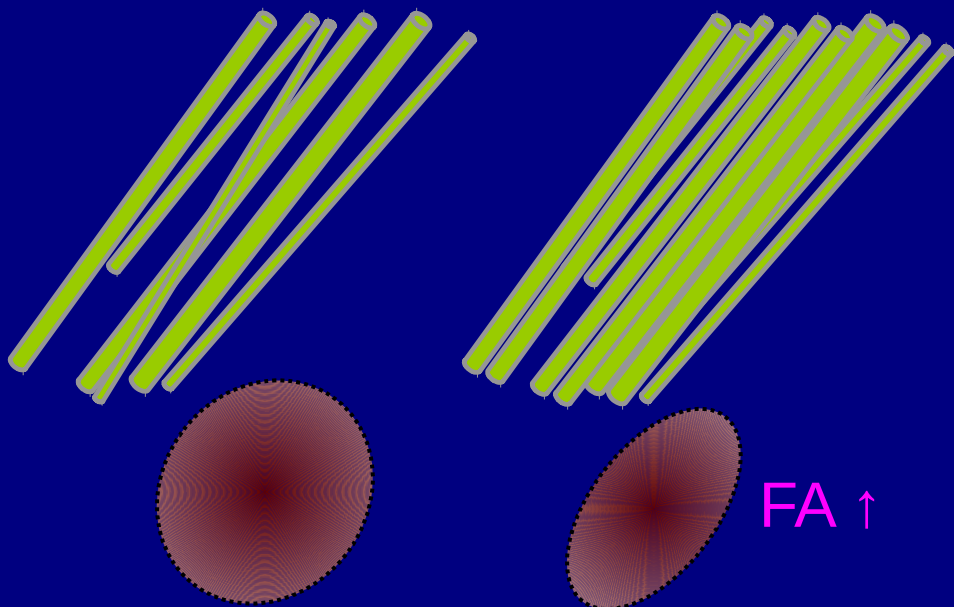
GM vs WM



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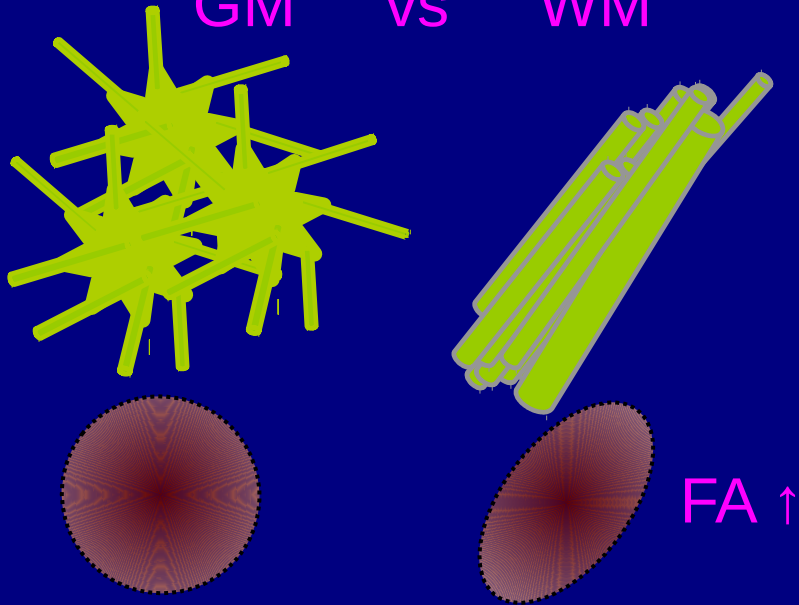


WM bundle density

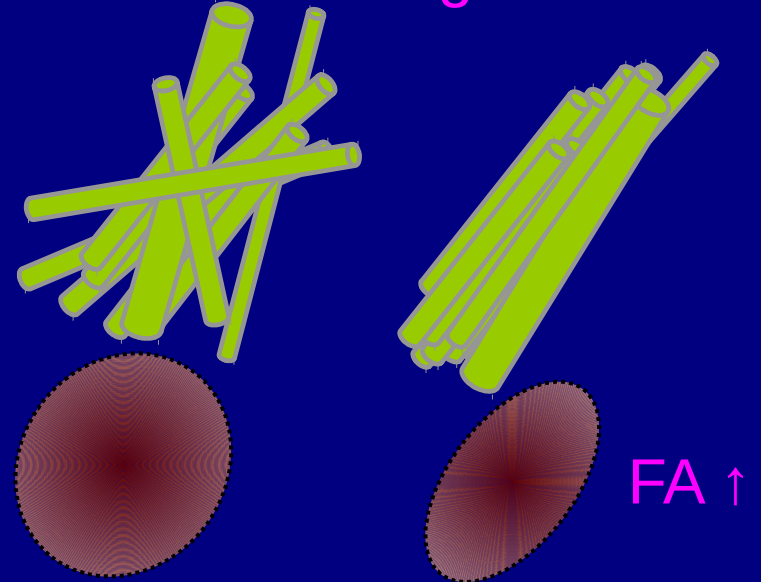


Cartoon examples: white matter \leftrightarrow FA

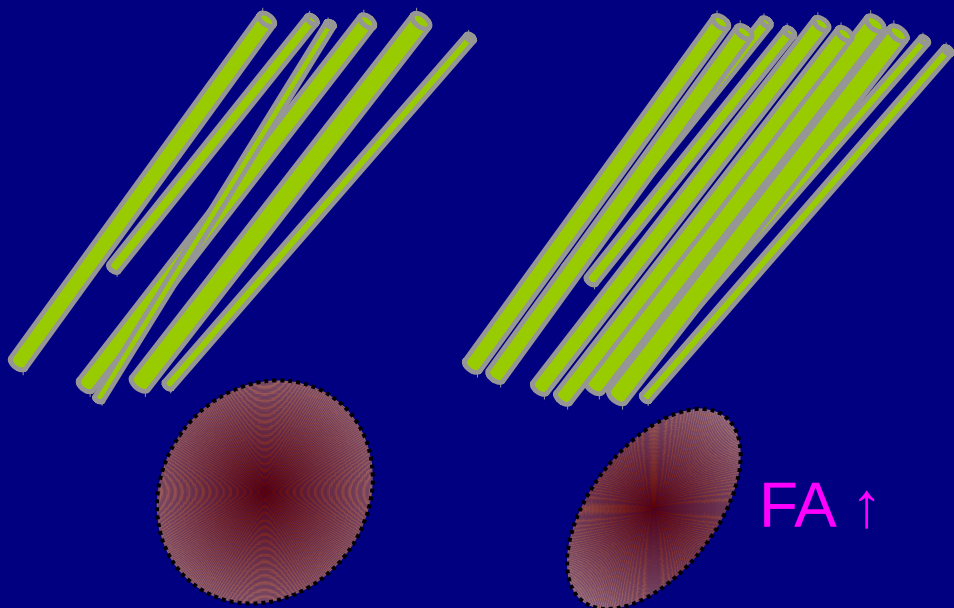
GM vs WM



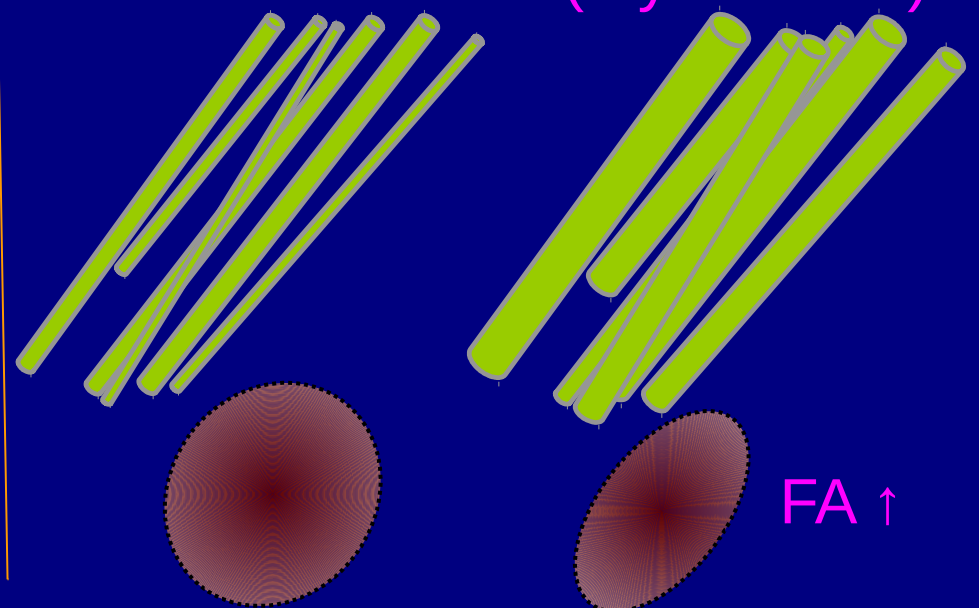
WM bundle organization



WM bundle density



WM maturation (myelination)



Interpreting DTI parameters

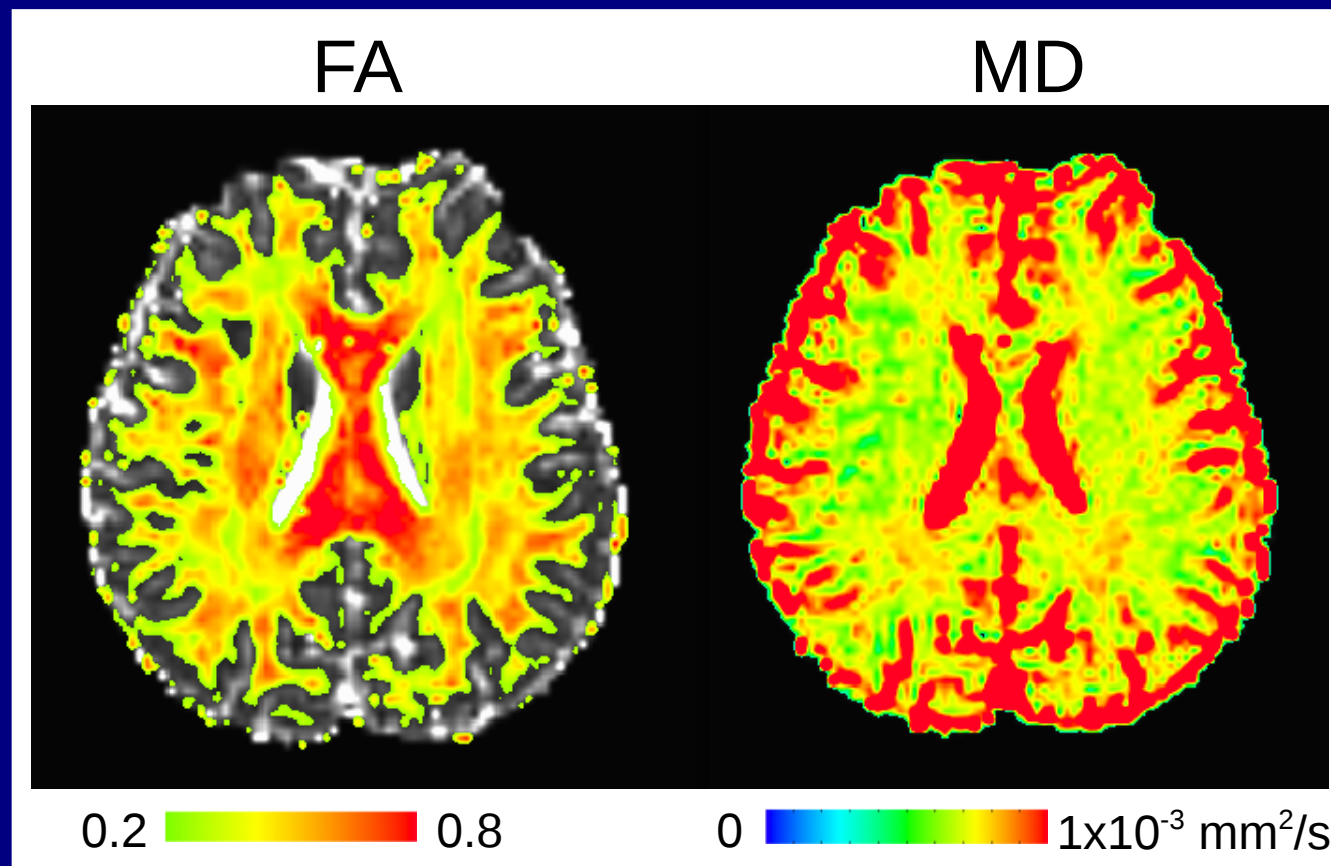
General literature:

FA: measure of fiber bundle coherence and myelination

- in adults, $FA > 0.2$ is proxy for WM

MD, L1, RD: local density of structure

e_1 : orientation of major bundles



Interpreting DTI parameters

General literature:

FA: measure of fiber bundle coherence and myelination

- in adults, $FA > 0.2$ is proxy for WM

MD, L1, RD: local density of structure

e_1 : orientation of major bundles

Cautionary notes:

- Degeneracies of structural interpretations
- Changes in myelination may have small effects on FA
- WM bundle diameter \ll voxel size
 - don't know location/multiplicity of underlying structures
- More to diffusion than structure-- e.g., fluid properties
- Noise, distortions, etc. in measures

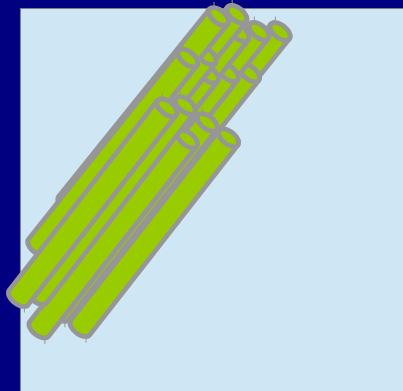
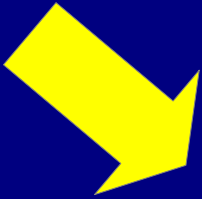
Acquiring DTI data:
diffusion weighted gradients in MRI

Diffusion weighted imaging

For a given voxel, observe relative diffusion along a given 3D spatial orientation (gradient)

DW gradient

$$\mathbf{g}_i = (g_x, g_y, g_z)$$

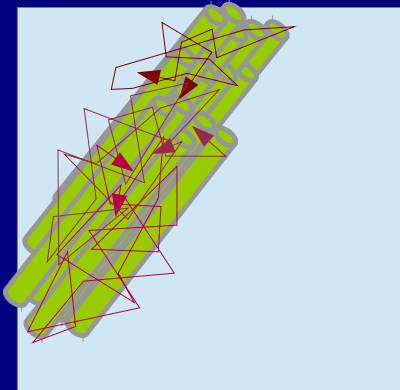


Diffusion weighted imaging

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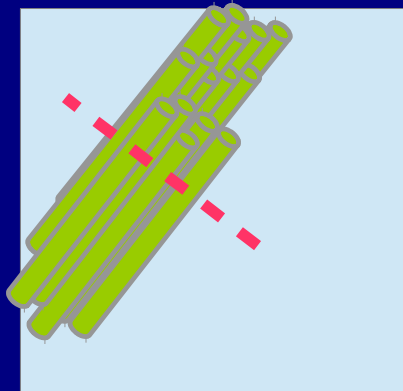
DW gradient

$$\mathbf{g}_i = (g_{x'}, g_{y'}, g_{z'})$$

MR signal is attenuated by diffusion throughout the voxel in that direction:

$$S_i = S_0 e^{-b \mathbf{g}_i^T \mathbf{D} \mathbf{g}_i}$$

→ ellipsoid equation of diffusion surface:
 $\mathbf{C} = \mathbf{r}^T \mathbf{D}^{-1} \mathbf{r}.$



Diffusion weighted imaging

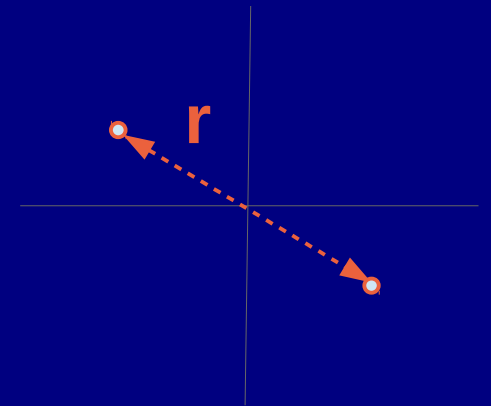
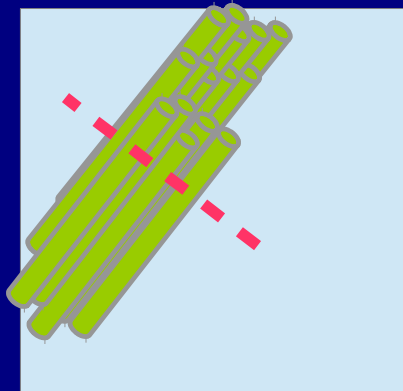
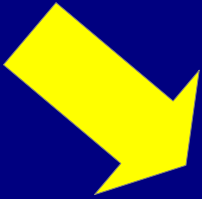
For a given voxel, observe relative diffusion along a given 3D spatial orientation (gradient)

DW gradient

$$\mathbf{g}_i = (g_x, g_y, g_z)$$

diffusion
motion
ellipsoid:

$$C_2 = \mathbf{r}^T \mathbf{D}^{-1} \mathbf{r}.$$



Diffusion weighted imaging

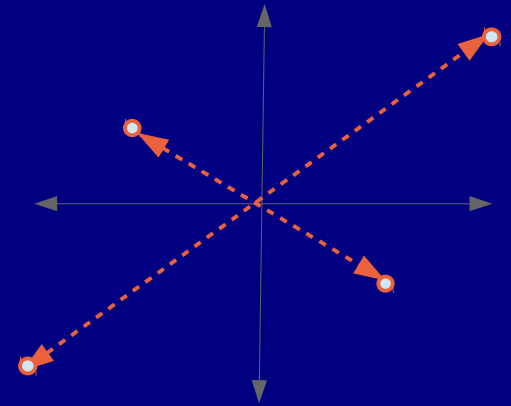
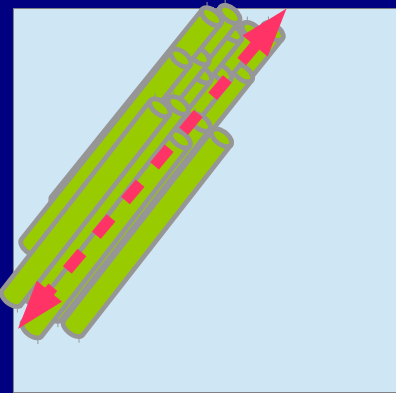
For a given voxel, observe relative diffusion along a given 3D spatial orientation (gradient)

DW gradient

$$\mathbf{g}_i = (g_x, g_y, g_z)$$

diffusion
motion
ellipsoid:

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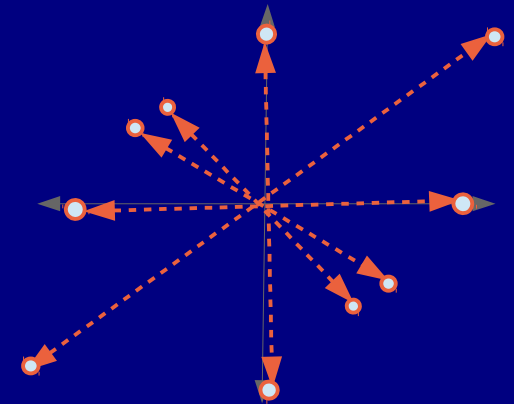
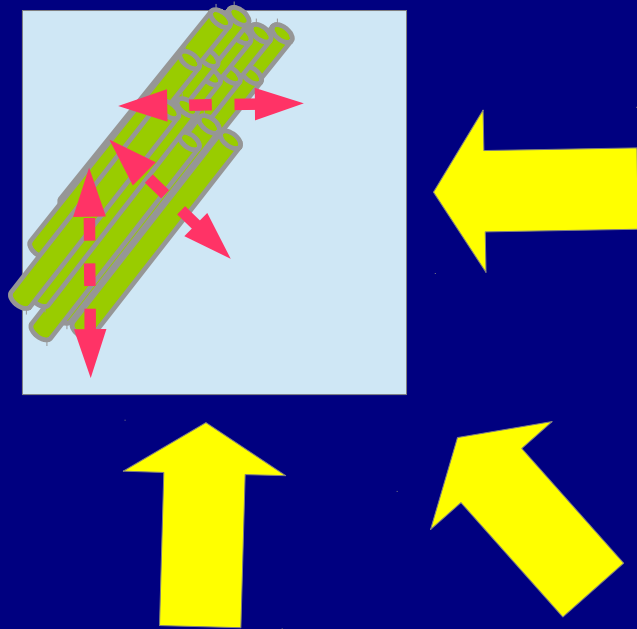
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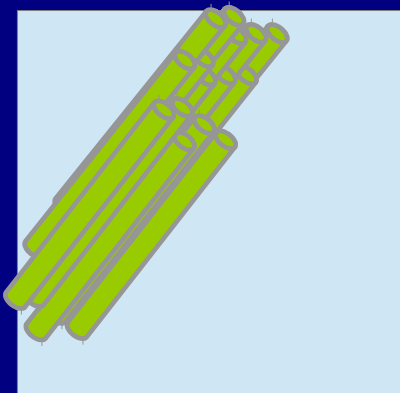


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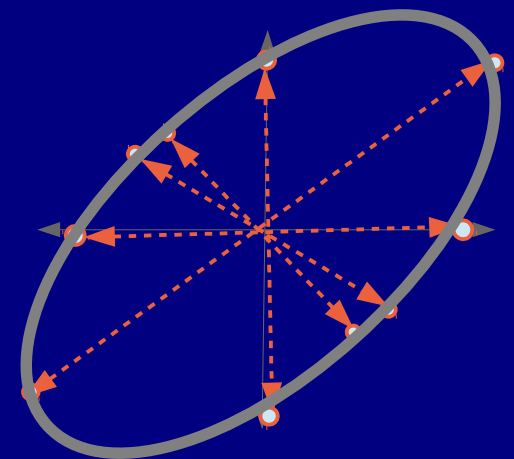


diffusion

motion

ellipsoid:

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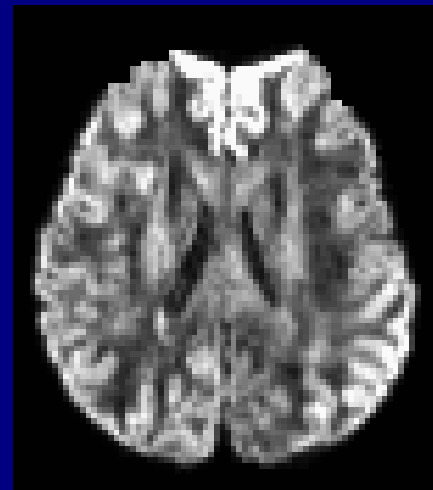
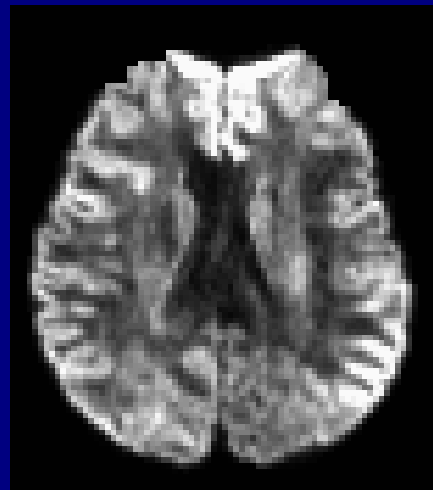
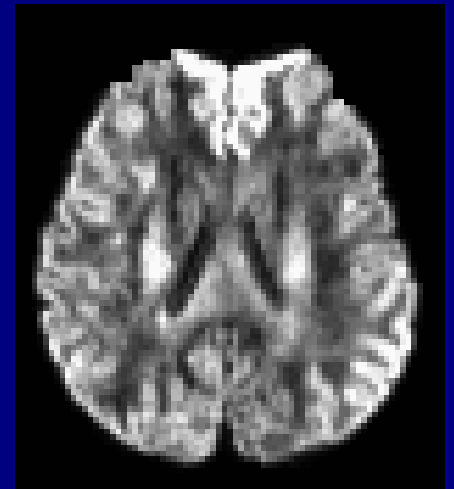
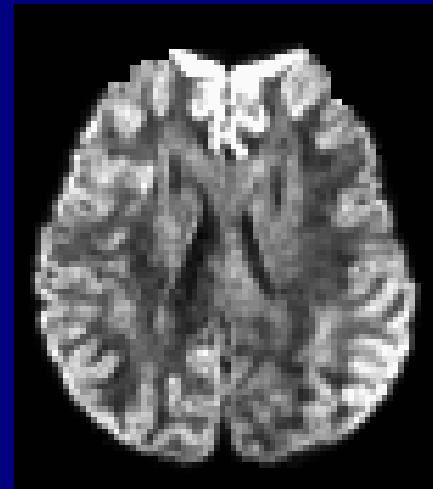
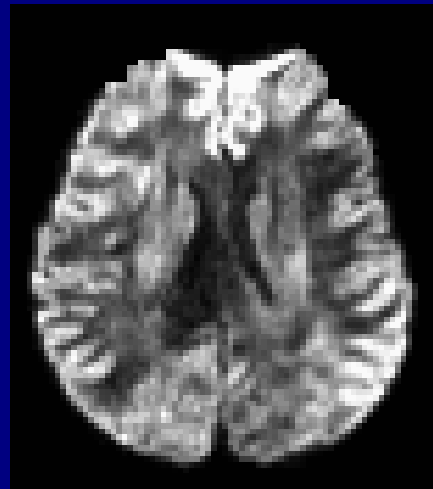
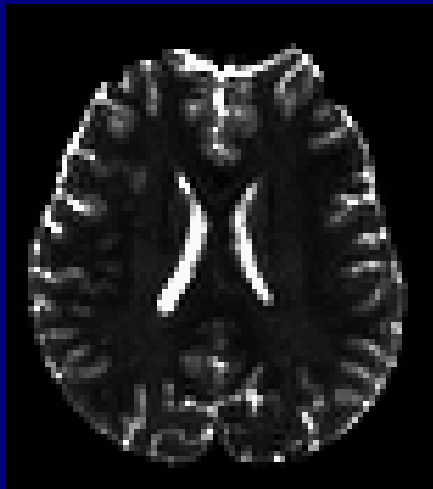
Individual points → Fit ellipsoid surface

Individual signals → Solve for \mathbf{D}

Sidenote: what DWIs look like

Unweighted
reference
 $b=0 \text{ s/mm}^2$

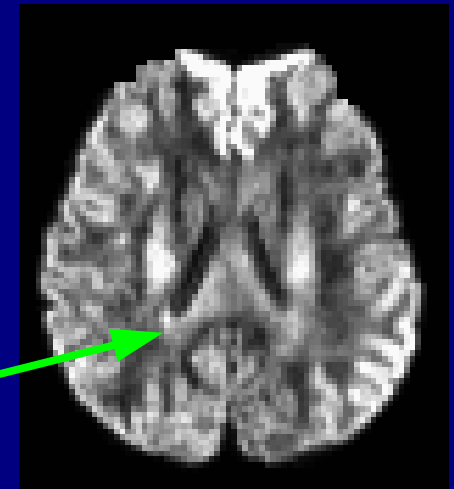
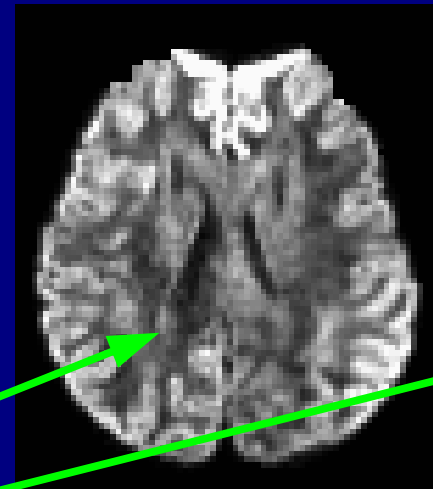
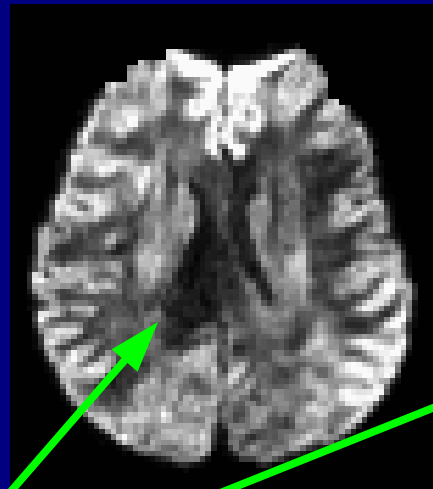
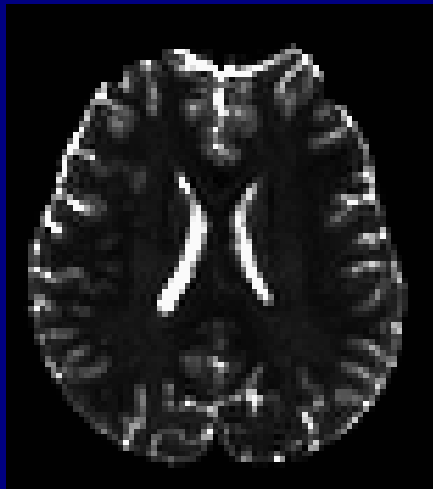
Diffusion weighted images
(example: $b=1000 \text{ s/mm}^2$)



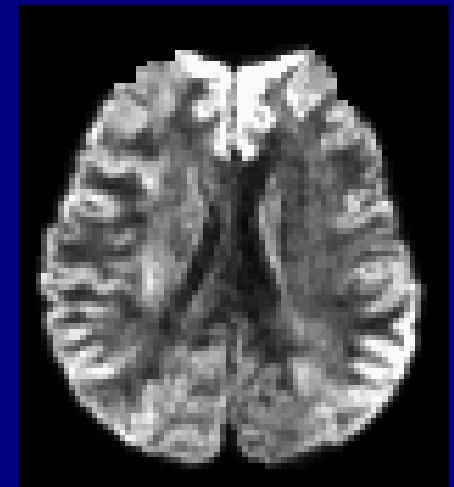
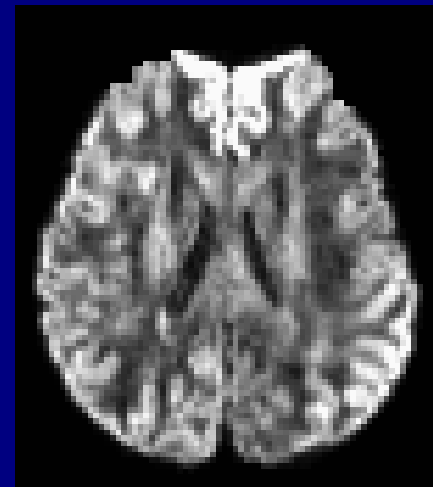
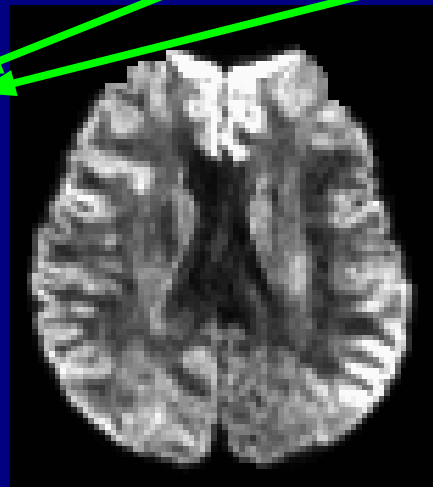
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Unweighted
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 $b=0 \text{ s/mm}^2$

Diffusion weighted images
(example: $b=1000 \text{ s/mm}^2$)



(Each DWI has a
different brightness
pattern: viewing
structures from
different angles.)



Noise in DW signals

MRI signals have additive noise

$$S_i = S_0 e^{-b \mathbf{g}_i^T \mathbf{D} \mathbf{g}_i} + \varepsilon,$$

where ε is (Rician) noise.

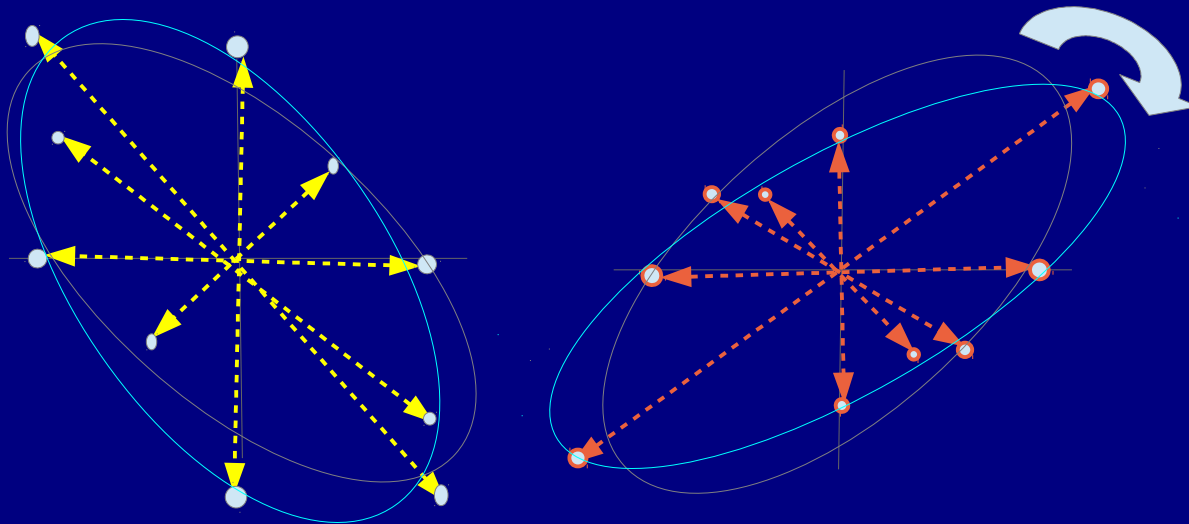
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→ Leads to errors in surface fit, equivalent to *rotations* and *rescalings* of ellipsoids:



'Un-noisy' vs perturbed/noisy fit

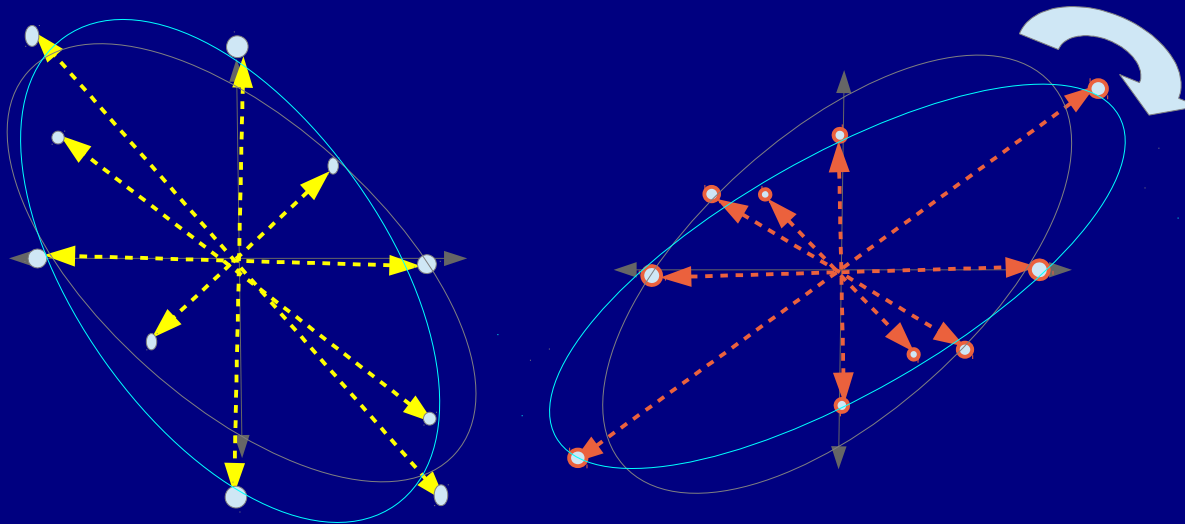
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'Un-noisy' vs perturbed/noisy fit

Leads to standard:
+ 30 DWs (~12 clinical)
+ repetitions of $b=0$
+ DW b chosen by:
 $MD * b \approx 0.84$
+ nonlinear tensor fitting

Distortions in DWI volumes

There are also **serious** sources of distortion when acquiring DWIs:

- + Subject motion

 - due to movement during/between volume acq. -> signal loss/overlap

- + Eddy current distortion

 - due to rapid switching of gradients -> nonlinear/geometric distortions

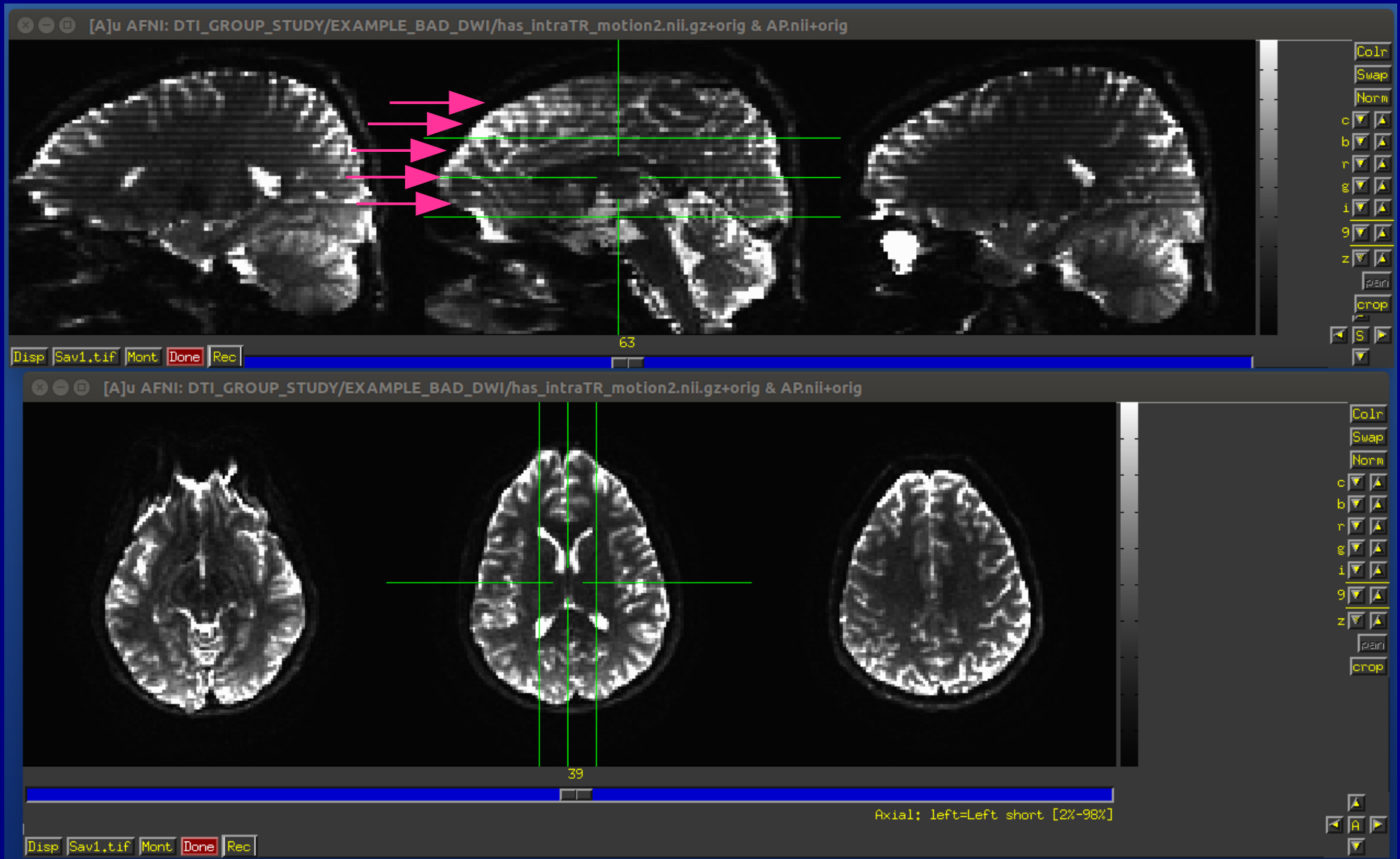
- + EPI distortion

 - due to B0 inhomogeneity -> geometric distortions along phase encoding dir, signal pileup or attenuation

---> And effects combine! Need careful acquisition (sometimes perhaps even **reacquisitions**) and post-processing.

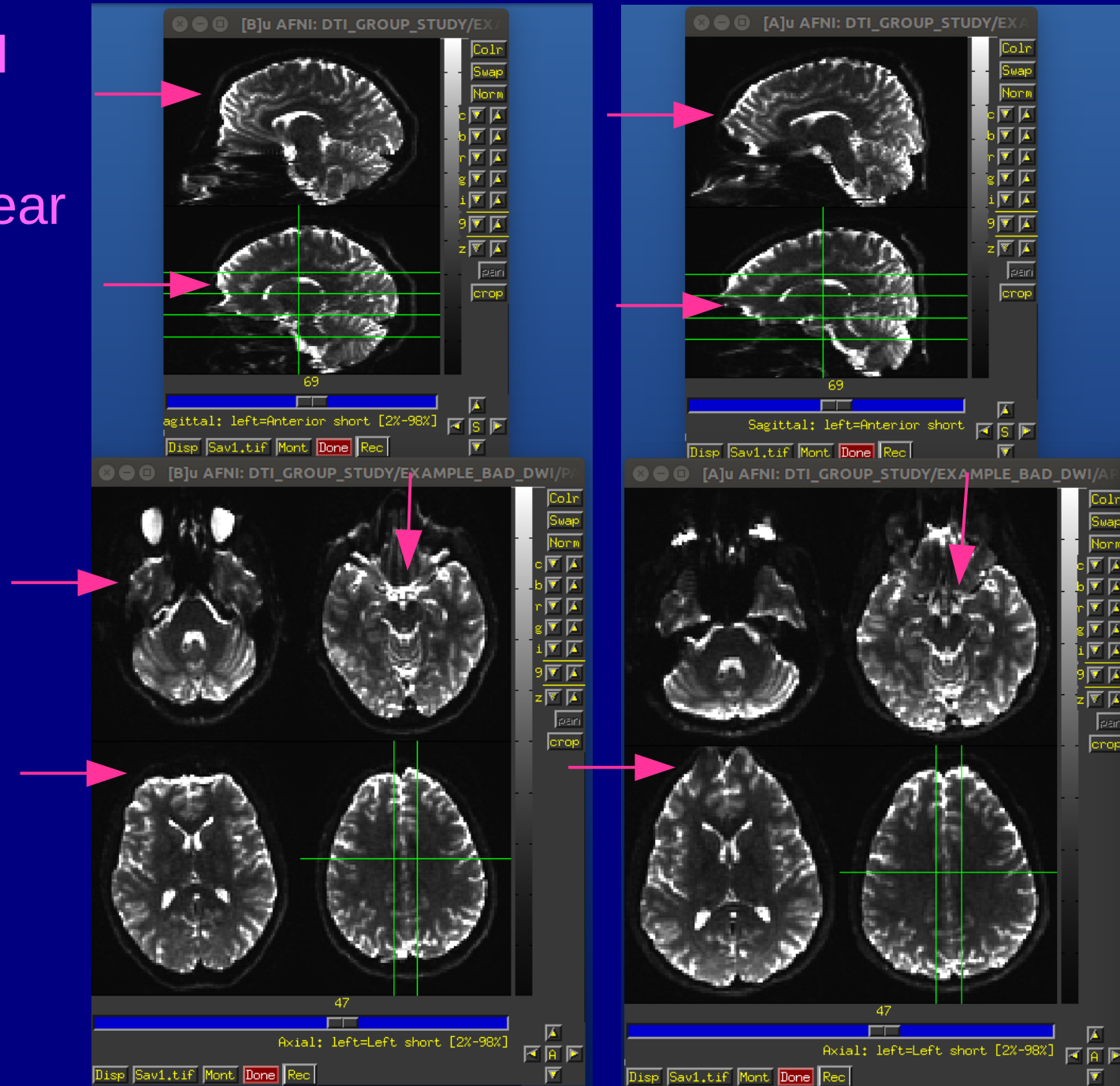
Distortions in DWI volumes

From subj motion: interleaved brightness distortions



Distortions in DWI volumes

From eddy and EPI distortions:
+ geometric/nonlinear warping
+ signal pileup and attenuation



SUMMARY

- + Diffusion-based MRI uses application of magnetic field gradients to probe the relative diffusivity of molecules along different directions.
- + DTI combines that information into a simple shape family, spheroids, to summarize the diffusivity.
- + From the DT, several useful properties are described in terms of scalar (e.g., FA, MD, L1) and vector (e.g., V1) parameters.
- + Many “standard” interpretations of DTI parameters exist (i.e., higher FA = “better” WM), but we must be cautious.
- + Distortions and noise affect all DTI estimates, and we must consider the consequences of these in all analyses.

