

WARPS

$$I(\underbrace{x + w(x)}_{\text{displacement}}) \approx \underbrace{B(x)}_{\text{base image}}$$

How to think about warps:

- For each location in base image (x), reach out to $x + w(x)$ to grab $I()$ and bring that value back
- It's not that $x + w(x)$ says where to put $I(x)$, it's that it says where to get the value that belongs at x
- This is what `3dQwarp` computes & what `3dNwarpApply` does with a warp

Programs

- `3dQwarp`
- `3dNwarpApply`
 - can auto-catenate warps & matrices
- `3dNwarpCat` - catenation
- `3dNwarpCalc` - more complex calculations
- `3dNwarpFores` - functions of a warp, like volume distortion at each voxel

UNWARPING

① Via Field Map

EPI distorted in phase encoding "y" direction

$$I(x, \underbrace{y + c\lambda(x,y)}_{\text{warping}}) = \frac{1}{1 + c \frac{\partial \lambda(x,y)}{\partial y}} \underbrace{M_{\perp}(x,y)}_{\text{true image}}$$

$\lambda(x,y)$ = frequency shift (say in Hz)

c = constant

$$= \tau \cdot \text{FOV}_y$$

? \longrightarrow τ = time between k_y lines (eg, $\approx 300 \mu\text{sec}$)

0018, 1310, 1312 \longrightarrow FOV_y = Field of view in y-direction (e.g. $\approx 240 \text{ mm}$)

$$\text{so } c \approx 3 \cdot 10^{-4} \text{ s} \cdot 240 \cdot 10^3 \text{ mm} = 0.072 \text{ mm/Hz}$$

Measured field map

Ⓐ Is it undistorted (measured by GRE) } determines if we have
or distorted itself (measured by EPI) } $\lambda(x,y)$ or a distorted $\lambda(x,y)$

Ⓑ contains a lot of noise and is unreliable near edges of brain - where most distortion is

Ⓒ so needs to be smoothed and extended outside brain where it was measured

\longrightarrow How to do (c) properly is the issue!

Given Ⓐ and (c), `3dNwarpApply` can transform a dataset.

UNWARPING

② Blip \rightarrow up/down (or left/right)

EPI pairs with scan 1 way ($c > 0$)
and the other way ($c < 0$)

$$\begin{aligned} I_p(x, y + c\lambda(x, y)) &\cong M(x, y) \\ I_m(x, y - c\lambda(x, y)) &\cong M(x, y) \end{aligned} \quad \left. \begin{array}{l} \text{unknown} \\ \text{"true" image} \end{array} \right\}$$

So use 3D warp with the "-plusminus" option to match

$$I_p(\underline{x} + \underline{w}(\underline{x})) \cong I_m(\underline{x} - \underline{w}(\underline{x}))$$

instead of the usual

$$I(\underline{x} + \underline{w}(\underline{x})) \cong B(\underline{x})$$

Then can use the "meet in the middle" warp $\underline{w}(\underline{x})$ to warp all blip up EPIs to the middle (and $-\underline{w}(\underline{x})$ to warp all blip down EPIs)

Properly done, blip up/down should scan both k_x & k_y backwards, and excite with opposite k_z gradients. Then all 3D distortions are reversed & 3D warp is reasonable

Advantages over fieldmap:

- no need to find (or fit) c or (c) [or even phase encode dir]
- 3D adjustment