

Multiple Comparisons: Embracing Instead of Fighting!

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Handout: [afni26_ROI-based-modeling.pdf](#)



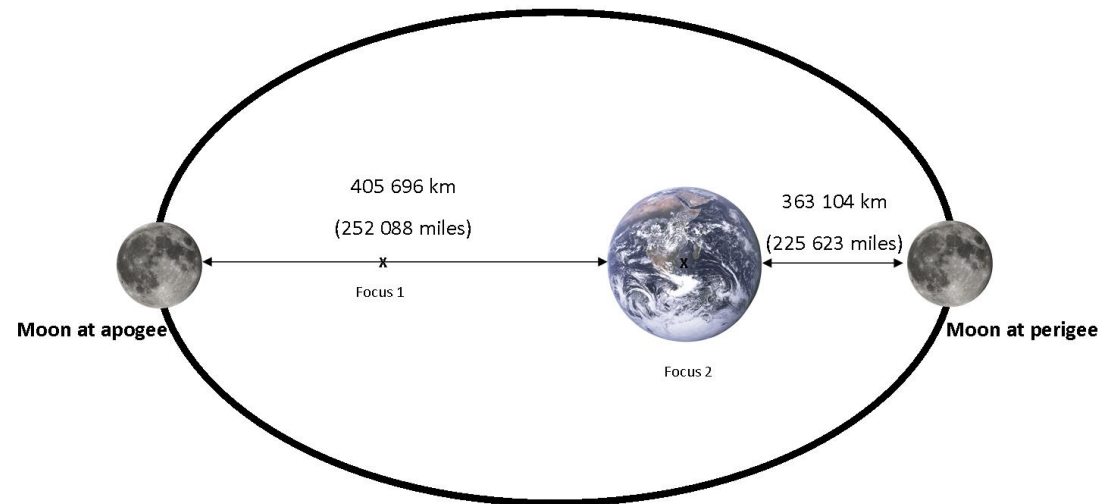
Preview

- **Current correction methods for multiplicity**
- **3 perspectives**
 - NHST: p -value and thresholding
 - Model accuracy
 - Integrative modeling
 - 2 toy examples: NBA players; Kidney cancer
- **Application: region-based analysis (RBA)**
 - Program in **AFNI**: **RBA**
- **Other applications**
 - Matrix-based analysis (program in **AFNI**: **MBA**)
 - Region-based inter-subject correlation (ISC) analysis
 - Gray matter connectivity analysis (DTI)
 - Other cases involving multiplicity

Reproducibility: start with physics

• What is the distance between earth and moon?

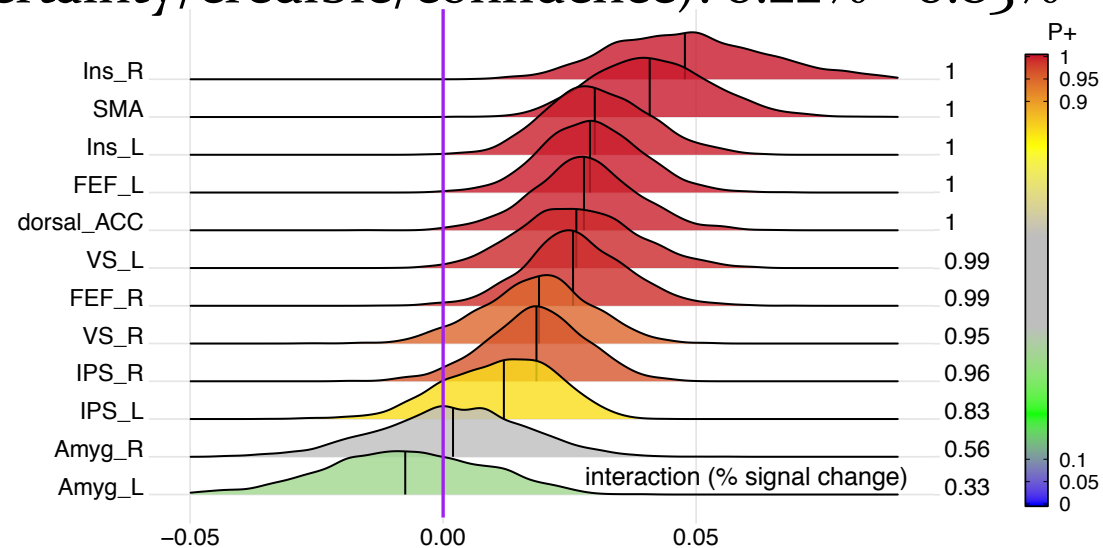
- t -statistic = 4.25 (or p -value = 0.01): informative?
- Ridiculous? Check out colorbars, tables and network graphs in publications/slides/posters...
- Average: 384,400 km
- Uncertainty:
363,104 km – 405,696 km



Reproducibility: neuroimaging

• What is the BOLD response in a brain region?

- t -statistic = 4.25 (or p -value = 0.01): informative?
- Colorbars, tables and network graphs in publications/slides/posters...
- Average: 0.52% signal change
- Range (uncertainty/credible/confidence): 0.22% - 0.83%



Multiplicity: correctness in the eye of beholder

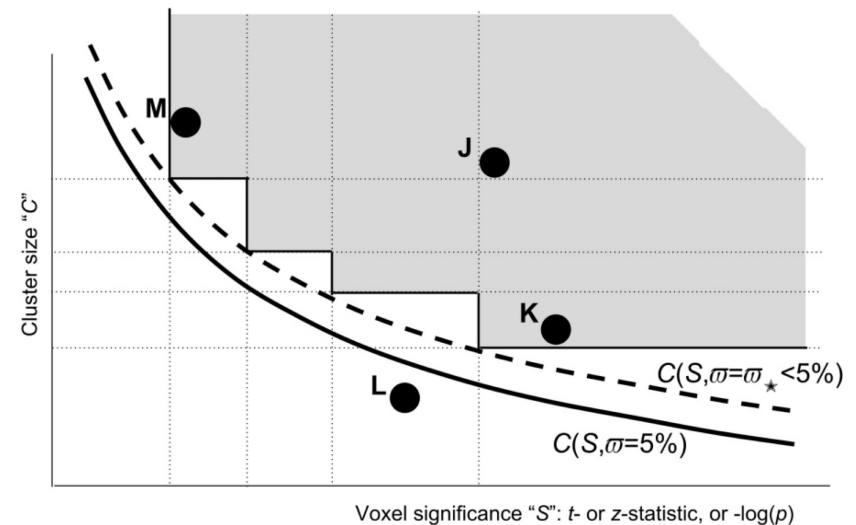
• 100,000 spatial units - 100,000 models: MUA

- Assumption of spatial independence
- Sharing no information

• Corrections

- Multiplicity + spatial relatedness
- Heavy penalty: information waste
- Arbitrariness

- Why not 0.04 or 0.06 instead of 0.05?
- Different correction methods: arbitrary voxel p vs. power
- Heavily dependent on data space: whole brain, gray matter, ROIs
- Information waste at global level: only local relatedness considered



Research reproducibility

- **Does strength of statistical evidence shrink?**

- Previous claim with statistical evidence: p -value = 0.03
- Current study with evidence: p -value = 0.04.
- Multiple testing issue? Should one adjust for multiplicity?
- How about all studies that use statistical analyses?

- **How are study repetitions distributed?**

- Same experiments repeated 100 times
- An effect (population, BOLD) across entities (counties, brain regions)

Null Hypothesis Significance Testing

- **Straw man H_0 : null hypothesis**

- Witch hunt: Don Quixote's windmills
- **Type I error** = $P(\text{data} \mid H_0) = \text{false positive} = p\text{-value}$
 - Surprise or weirdness of data: **0.05**
 - No effect until shown with small **p -value**
 - Innocent until proven guilty
- **Type II error** = $P(\text{accept } H_0 \mid H_1) = \text{false negative}$



- **Real practice: type I error ONLY**

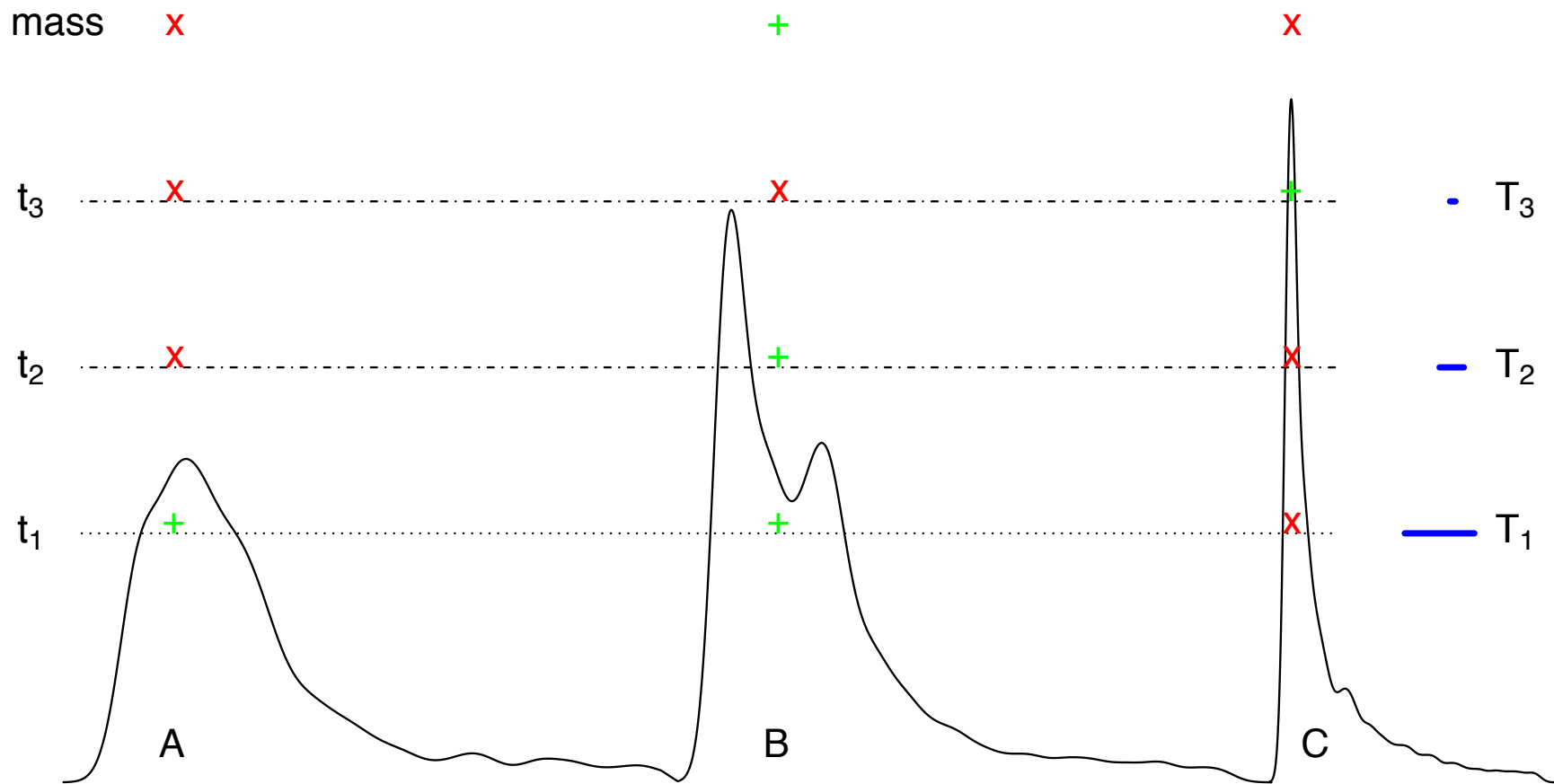
- False positives: purely pleasing to statisticians!
- With NO regard for type II error

	H_0 True	H_0 False
Reject H_0	Type I Error (false positive)	Correct
Fail to Reject H_0	Correct	Type II Error (false negative)

Results interpretation

- What is the conclusion of a region where $p=0.6$?
- If $p=0.05$, what is the probability for the region being activated?

Clusters vs islands: arbitrariness

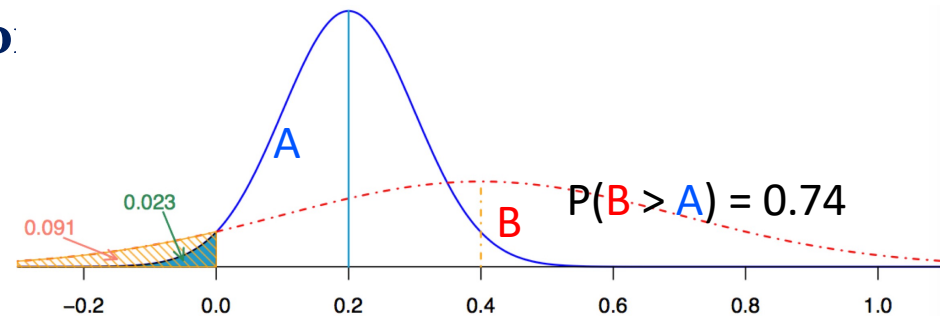


Issues: NHST

- **Arbitrary dichotomy:** where to draw a line in the sand?
 - Binary or discrete: innocent vs guilty
 - p -value itself is a random variable
 - Unrealistic: “activated” vs “not activated”?
 - Methods for correlation matrix: why is 0.3 so special?
- **Vulnerable to misconceptions**
 - $p(\text{weirdness} | H_0) \neq p(H_0 | \text{data})$
 - Absence of evidence \neq evidence of absence
- **Vulnerable to data manipulations**
 - Statistical evidence changes: whole brain, gray matter, region
- **Inflated effect estimates**
 - Type M (magnitude) and type S (sign) errors: biasedness

Issues: NHST

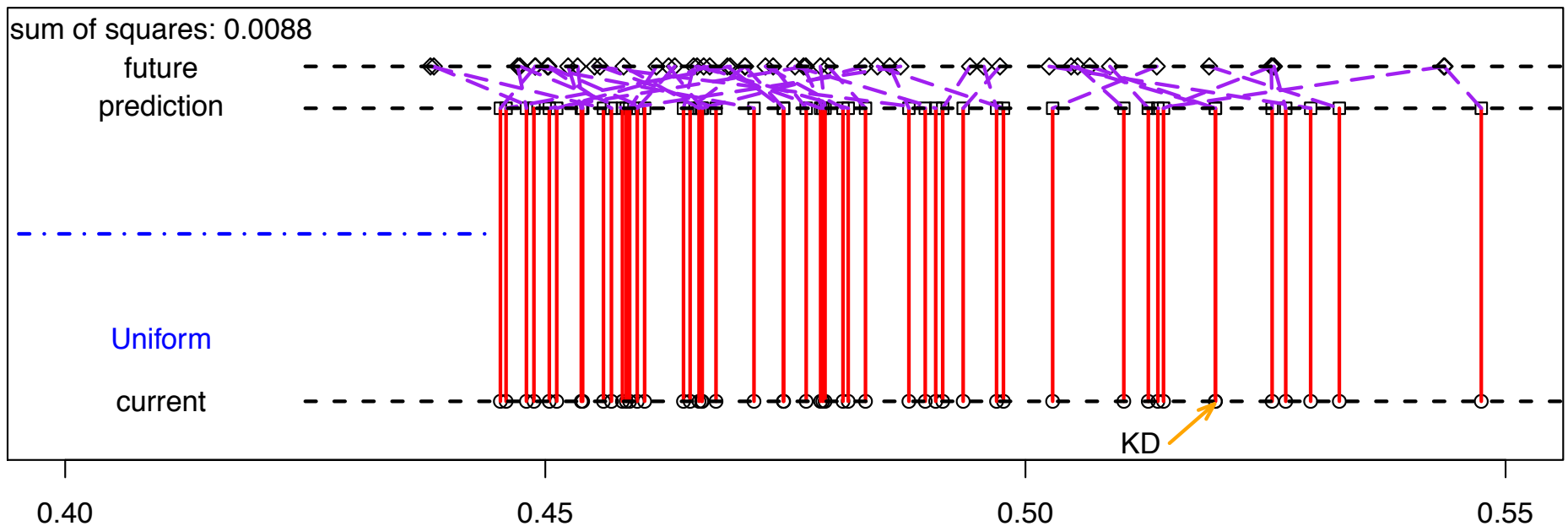
- **Inefficient modeling**
 - Over-penalizing
 - Ignore false negative (power)
 - No mechanism to incorporate prior knowledge
- **Disregarding effect size**
- **Uncertainty unavailable**
 - No standard deviation at voxel
- **Lack of spatial specificity**
 - Locating regions per peak voxel
- **Penalizing small regions**



Toy Example 1

- **NBA players**

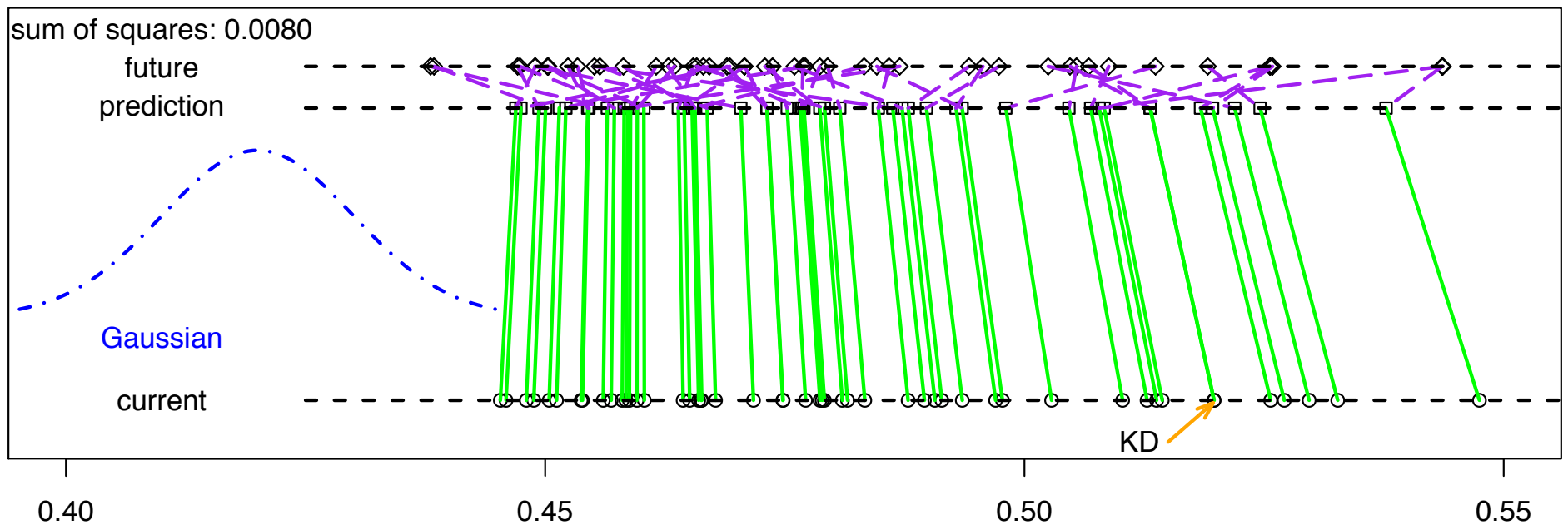
- Kevin Durant field goals percentage: 52.1%
- Prediction: performance during next season?
- One vs. top 50 players: **no pooling** vs complete pooling



Toy Example 1

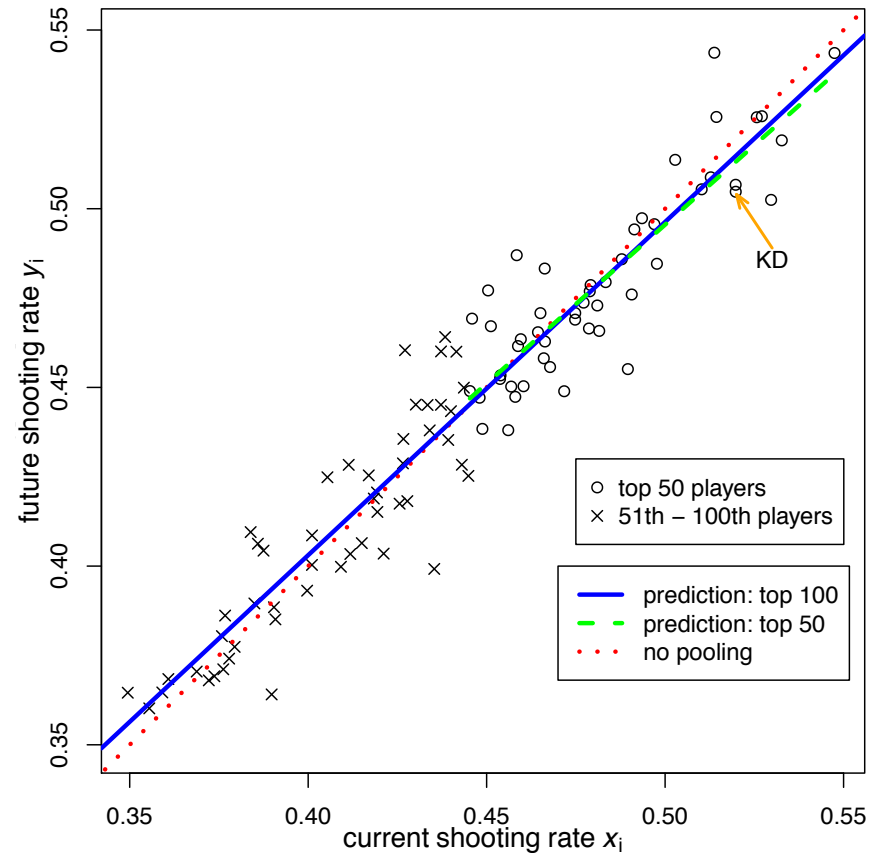
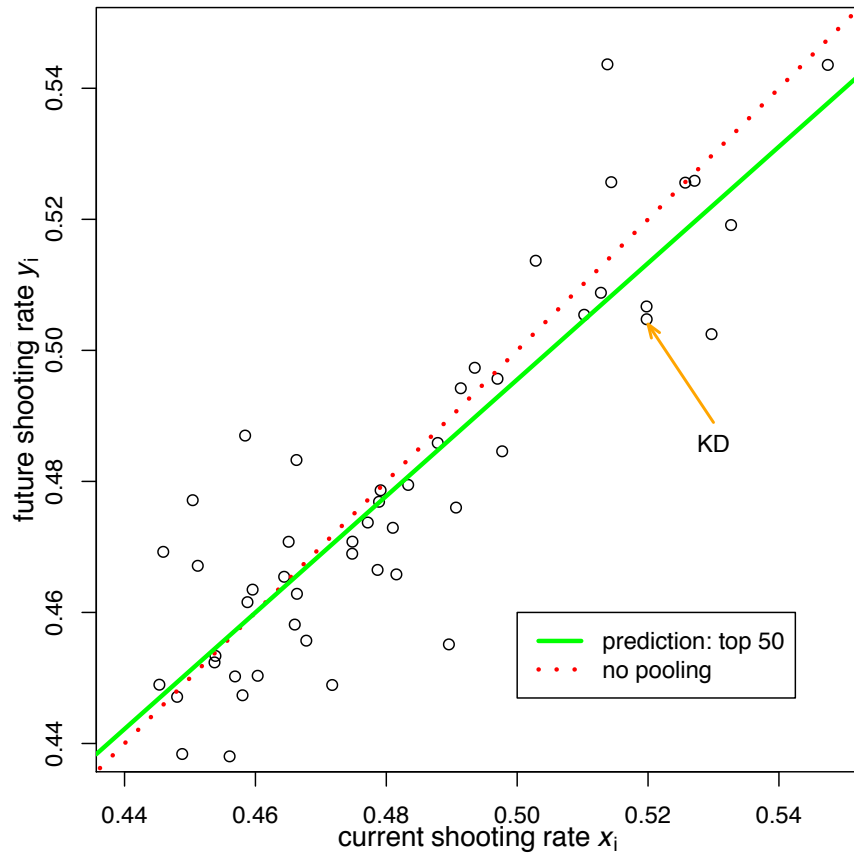
- **NBA players**

- Kevin Durant field goals percentage during 2019: 52.1%
- Prediction: performance during 2020?
- One vs. top 50 players: **partial pooling** (regression to the mean)



Toy Example 1

- Top 50 vs. 100 NBA players: **adaptivity**



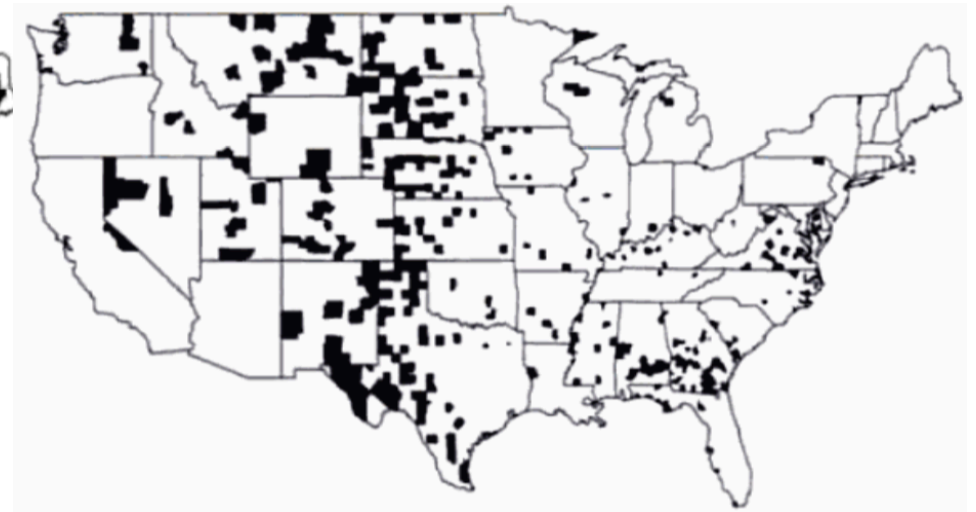
Toy Example 2

- Kidney cancer distribution among U. S. counties

Highest rate



lowest rate



Calibration

Morals from kidney cancer data

- **Multiplicity problem: > 3000 counties!**
 - Divide p -value by number of counties?
 - Borrow idea from neuroimaging: leverage geographical relatedness?
- **What can we learn from the example? Food for thought**
 - Care about strawman H_0 (zero kidney rate), false positives, p -value?
 - Trust individual county-wise estimates? **Unbiased! BLUE**
 - **Incorrect sign errors** (type S): some counties really have higher kidney cancer rate than others?
 - **Incorrect magnitude** (type M): some counties really have higher/lower cancer rate?
 - Would correction for multiplicity help at all?
 - Useless in controlling for type S and M errors
- **How can we do better?**
 - Information share: across spatial elements
 - **Research hypothesis: $P(\text{effect} > 0 \mid \text{data})$**

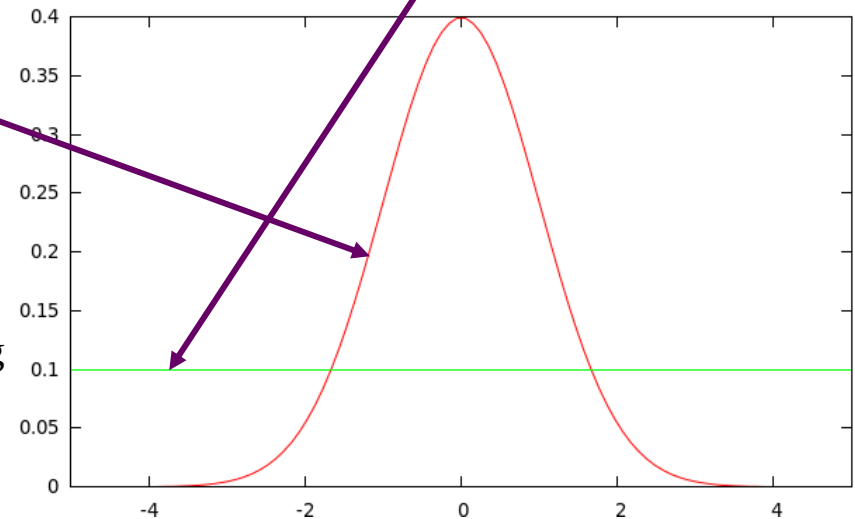
What do we know about spatial elements?

- **Massively univariate modeling**

- Pretend full ignorance: fully trust the data
- Uniform distribution: each element equally likely to have any value in $(-\infty, +\infty)$
- Similar for variances: variances can be negative in ANOVA

- **One crucial prior for spatial elements**

- Reasonable to assume Gaussian distribution?
- Gaussian assumption adopted everywhere!
 - Subjects, residuals across TRs
- How can Gaussian assumption help?
 - Loosely constraining elements
 - No full trust for individual estimates
 - Information sharing: shrinkage or partial pooling
 - Controlling type S and M errors



Short summary: **what we intend to achieve**

- **Abandon strawman and p -value**

- Directly focus on research interest: $P(\text{effect} > 0 \mid \text{data})$ vs. $P(\text{data} \mid \text{effect} = 0)$

- **Build one model**

- Incorporate all elements into a multilevel or hierarchical structure
- Loosely constrain elements: leverage **prior knowledge**
- Achieve higher modeling efficiency: **no more multiplicity!**
- Validate the model by comparing with potential competitors
- Be conservative on effect estimates by controlling type S and M errors: **biased?**
- Always be mindful of uncertainties: strength of evidence (no proof)
- Less vulnerable to data manipulations: whole brain, gray matter, regions, ...

- **Avoid dichotomous decisions**

- Report full results if possible
- Highlight instead of hide based on gradient of evidence
- Focus on estimation, not inferences

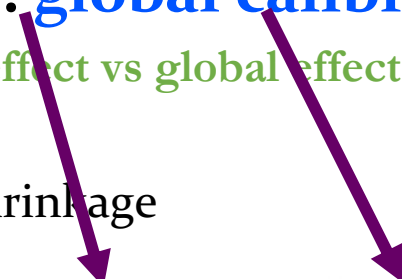
Bayesian strategy in handling multiplicity

- **Conventional approach: neighborhood leverage**

- Local relatedness: all regions act freely from each other

- **BML approach: global calibration**

- Tug of war: local effect vs global effect
- Weighted average
- Partial pooling, shrinkage


$$\hat{\theta}_j = \frac{\frac{n}{\lambda^2 + \sigma^2} \bar{y}_{\cdot j} + \frac{1}{\tau^2} b_0}{\frac{n}{\lambda^2 + \sigma^2} + \frac{1}{\tau^2}}, \quad V = \frac{1}{\frac{n}{\lambda^2 + \sigma^2} + \frac{1}{\tau^2}}$$

Application: region-based analysis

• Dataset

- Subjects: $n = 124$ children; resting-state data (Xiao et al., 2019)
- Individual subjects: seed-based correlation for each subject
 - 3D correlation between seed and whole brain (“functional connectivity”)
- Explanatory variable (behavior data): Theory of Mind Index x_i

• Voxel-wise group analysis: GLMs

- Focus: association between x and seed-based correlation (z-score)
- **Pretense**: voxels **unrelated** - equal likelihood within $(-\infty, \infty)$
- **Information waste!**
- GLMs: mass univariate - **multiplicity**
 $m = 100,000$ voxels \rightarrow
100,000 models

Xiao et al., 2019. [Neuroimage 184:707-716](#)

Uniform distribution:
total freedom - each
parameter on its own

1st voxel: $y_1 = a_1 + b_1 x + \epsilon_1$

2nd voxel: $y_2 = a_2 + b_2 x + \epsilon_2$

...

m th voxel: $y_m = a_m + b_m x + \epsilon_m$

GLMs: dealing with multiplicity!

- **Voxel-based analysis: GLMs**

- **Penalty time** for pretense: multiple testing ($m = 100,000$), magic **0.05**
- **Show time** for various correction methods
 - Voxel-wise p , FWE, FDR, spatial smoothness, clusters, ...
 - Simulations, random field theory, permutations, ...
 - How would dataset turn out under GLM? **4 lucky clusters** managed to survive

voxel p	cluster threshold	surviving ROIs	ROIs
0.001	28	2	R PCC, PCC/PrC
0.005	66	4	R PCC, PCC/PrC., L IPL, L TPJ
0.01	106	4	R PCC, PCC/PrC., L IPL, L TPJ
0.05	467	4	R PCC, PCC/PrC., L IPL, L TPJ

Switching from voxels to ROIs: **still GLMs**

- **Region-wise analysis : GLMs**

- Focus: association between and seed-based correlation (z-score)

- **Pretense**: ROIs **unrelated**

- GLMs: mass univariate

$m = 21$ ROIs \rightarrow
21 models

- **Penalty** for pretense:
multiple testing – what to do?

- **Bonferroni**? Unbearable
- What else?

Uniform distribution:
total freedom - each
parameter on its own.

1st ROI: $y_1 = a_1 + b_1 x + \epsilon_1$

2nd ROI: $y_2 = a_2 + b_2 x + \epsilon_2$

...

m th ROI: $y_m = a_m + b_m x + \epsilon_m$

Switching from GLMs to LME

- **Region-wise analysis : Linear Mixed-Effects (LME) model**

- One model integrates all regions
- ROIs loosely **constrained** instead of being **unrelated**
 - Gaussian distribution: Is it far-fetched or subjective?
 - Similar to cross-subject variability

- Goal: effect of interest- $a + \alpha_j, b + \beta_j$

- Differentiation: fixed vs. random

- Fixed: **epistemic** uncertainty
- Random: **aleatoric** uncertainty
- Julius Caesar: Alea iacta est. January 10, 49 BC

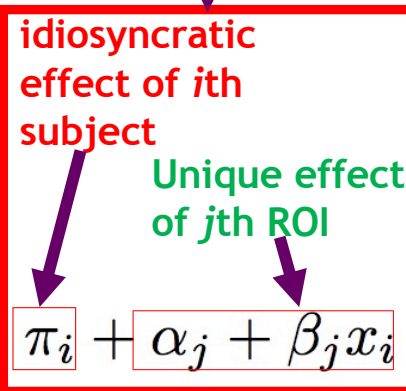
- What can we get out of LME?

- Conventional framework
- Estimates for fixed effects
- Variances for random effects

- **Dead end!**

Overall effect:
shared by all ROIs
and subjects

New components



$$z_{ij} = a + bx_i + \pi_i + \alpha_j + \beta_j x_i + \epsilon_{ij}$$

$$\pi_i \stackrel{iid}{\sim} \mathcal{N}(0, \tau^2), (\alpha_j, \beta_j)^T \stackrel{iid}{\sim} \mathcal{N}(\mathbf{0}, \boldsymbol{\lambda})$$

$$\epsilon_{ij} \sim \mathcal{N}(0, \sigma^2), i = 1, 2, \dots, n, j = 1, 2, \dots, m$$

Switching from GLMs to **BML**

- **Region-wise analysis : Bayesian multilevel (BML) model**

- **One** model integrates all regions: basically same as LME
- ROIs loosely **constrained** instead of being **unrelated**
 - Gaussian distribution: Is it far-fetched or subjective?
 - Similar to cross-subject variability
- **Goal: effect of interest $b + \beta_j$**
- No more differentiation: fixed vs. random
 - All parameters: **aleatoric**
- Same model as LME plus **priors**
 - **Markov chain Monte Carlo (MCMC)**
 - Inferences via posterior distribution
- **Ka-ching!**

New components

Idiosyncratic effect by i th subject

Unique effect by j th ROI

Overall effect: shared by all ROIs and subjects

$$z_{ij} = a + bx_i + \pi_i + \alpha_j + \beta_j x_i + \epsilon_{ij}$$

$$\pi_i \stackrel{iid}{\sim} \mathcal{N}(0, \tau^2), (\alpha_j, \beta_j)^T \stackrel{iid}{\sim} \mathcal{N}(\mathbf{0}, \boldsymbol{\lambda})$$

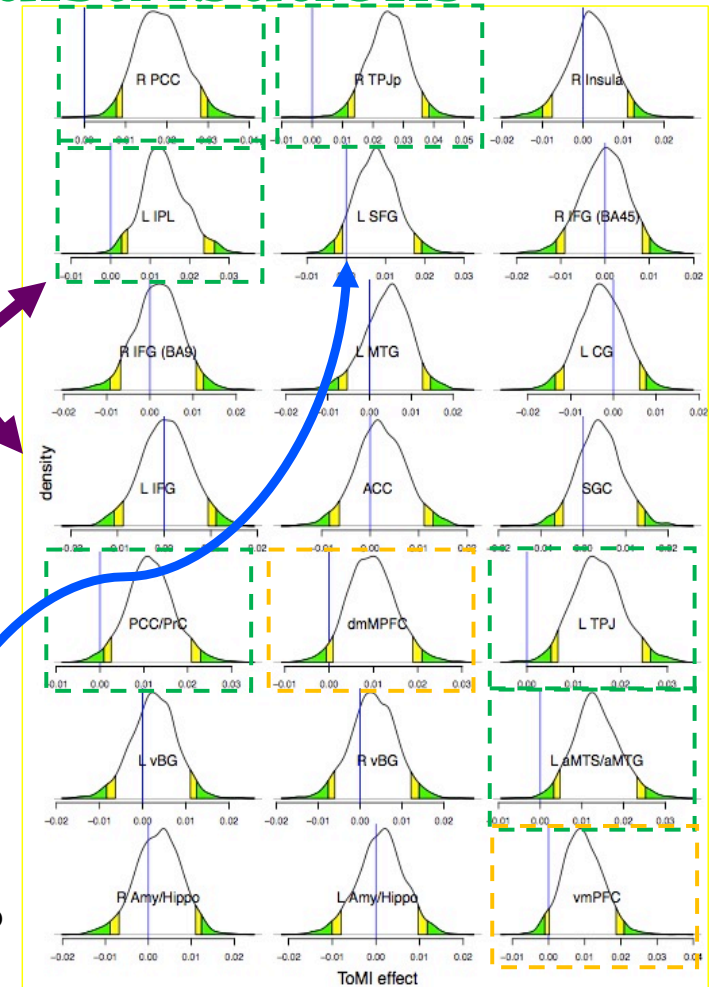
$$\epsilon_{ij} \sim \mathcal{N}(0, \sigma^2), i = 1, 2, \dots, n, j = 1, 2, \dots, m$$

Inferences from BML: full distributions

- Region-based BML: 21 ROIs
- Full report with richer information: posterior distributions for each ROI
 - No dichotomization
 - No results hiding
 - No discrimination against small regions
 - No ambiguities about spatial specificity
 - No inconvenient interpretation of confidence interval
 - Evidence for each ROI: $P(\text{effect} > 0 \mid \text{data})$
- 9 ROIs with strong evidence of effect compared to
 - Region-wise GLM with Bonferroni correction
 - Voxel-wise GLM at cluster level: 2 clusters

Highlight, not hide

How about Left SFG?



Inferences from BML: uncertainty

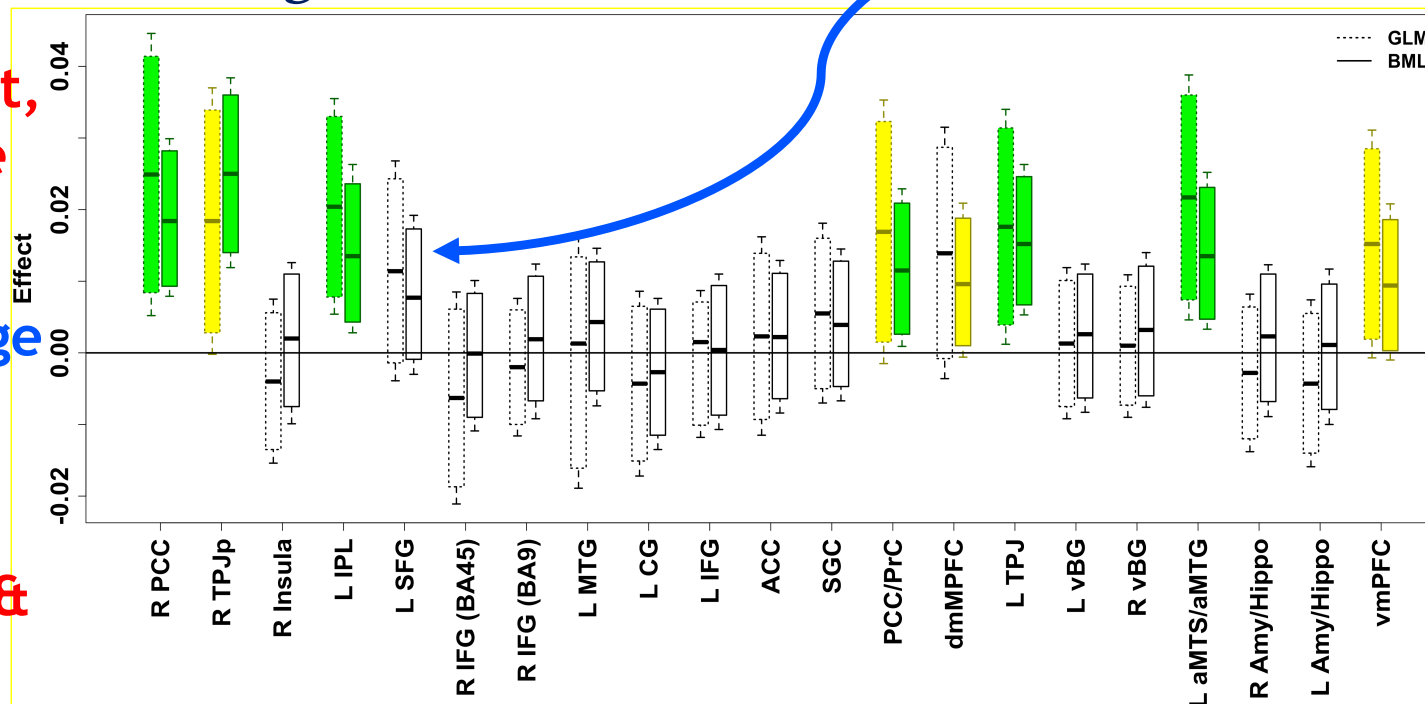
- ROI-based BML: 21 ROIs
- Full report with **bar graph** uncertainty intervals
 - **Nothing hidden under sea level**
- 8 ROIs with strong evidence for effect of interest

How about Left SFG?

Highlight,
not hide

Shrinkage
/ partial
pooling

Type M &
S errors



BML: model validations

- **Cross-validation**

- Leave-one-out information criterion (LOOIC)

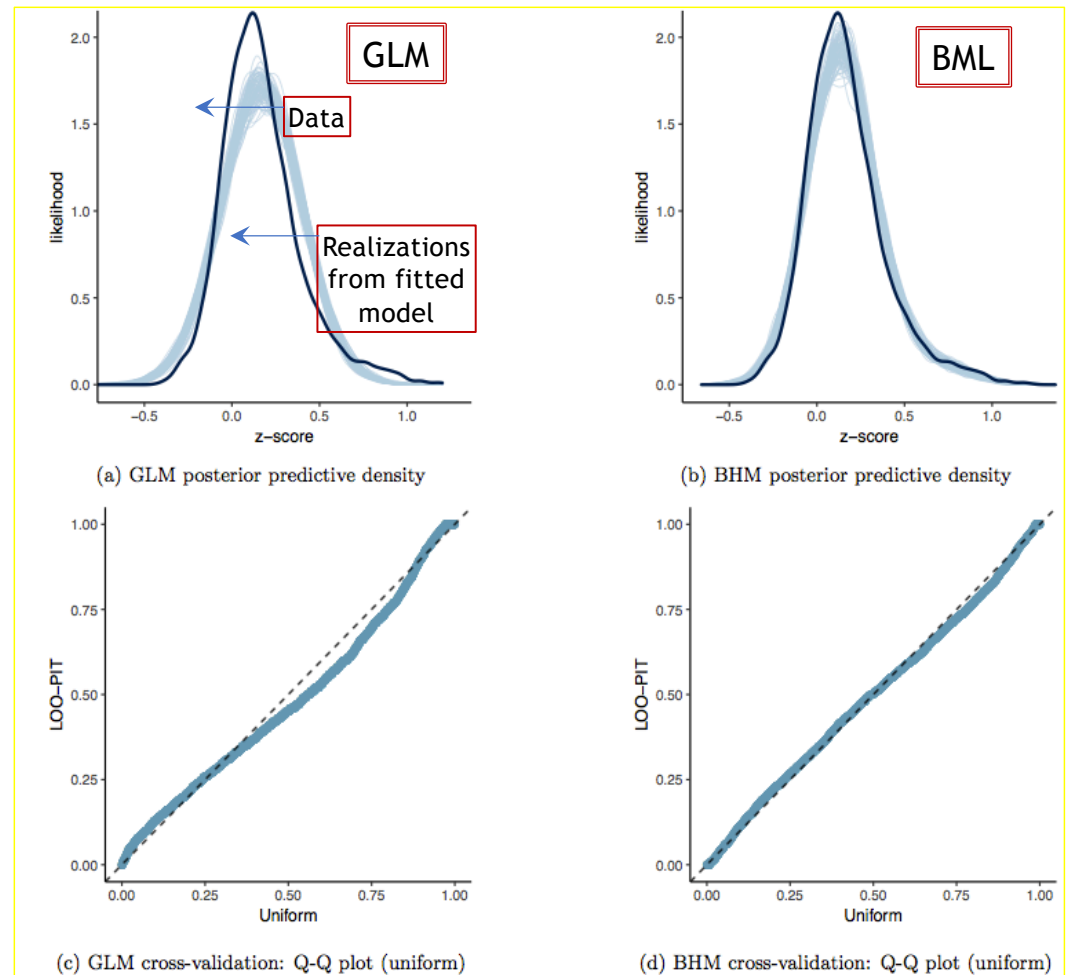
Cross-validation

	LOOIC	SE
GLM	-300.39	98.25
BML	-2247.06	86.42
GLM - BML	1946.67	96.35

- Posterior predictive checking

- **Effects of BML**

- Regularizing ROIs: don't fully trust individual ROI data
- Sacrificing fit at each ROI; achieving better overall fit



BML: Whole-brain vs. region-base analysis

- **Region-based analysis**

- + high region specificity: region definitions considered as priors
- + low computational cost
- + avoiding potential alignment issues by defining regions in native space: FreeSurfer + SUMA
- not all regions have been defined
- **information loss** due to averaging within each region
- region definitions can be tricky
 - relying on results accuracy in literature (e.g., publication bias)
 - different atlases/parcellations

- **Whole-brain analysis**

- + independent of region definitions
- + less likely to miss small regions that are not in available atlases/parcellations
- vulnerable to poor alignment across subjects
- region specificity problem
 - Voxel-wise results do not respect region definitions
- Computationally challenging
 - hopeful: within-chain parallelization and GPU usage

Application #2: matrix-based analysis

- **Dataset: correlation matrix**

- Subjects: $n = 41$ subjects; response-conflict task (Choi et al., 2012)
- Individual subjects: **correlation matrix** among $m = 16$ ROIs
- How to go about group analysis?
 - GLM for each element in correlation matrix: NBS, CONN, FSLNets in FSL, GIFT
 - Binarization approach: graph theory
- More broadly: matrix-based analysis (MBA) (“network modeling”)
 - **Inter-region correlation** (IRC): FMRI
 - **White matter properties** (FA, MD, ...): DTI
 - Other matrices (e.g., coherence, entropy, mutual information)

- **Focus on GLM**

- Student t -test or GLM on each element
 - $M = 120$ massively univariate models
- **Pretense** again: all elements are **unrelated**
- Equal likelihood within $(-\infty, \infty)$
- **Information waste**
- **Penalty time** again: permutations? FDR?

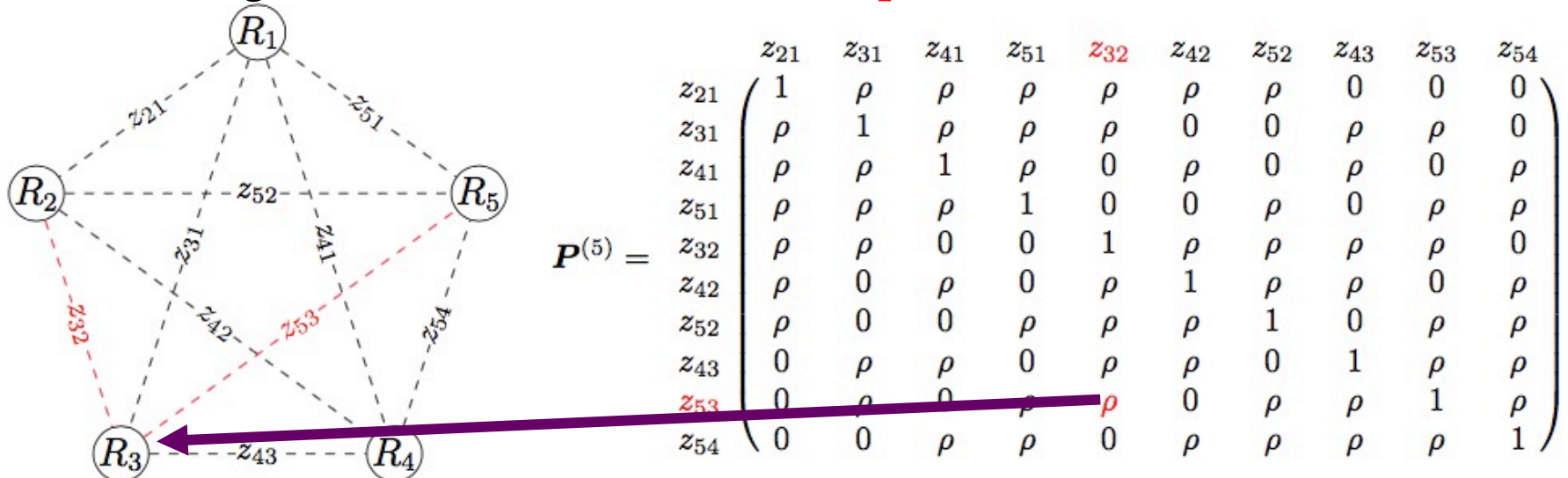
Choi et al., 2012. Neuroimage 59(2):1912-1923

$$Z_k^{(m)} = \begin{matrix} & R_1 & R_2 & R_3 & \cdots & R_m \\ R_1 & \left(\begin{array}{cccc} - & z_{12k} & z_{13k} & \cdots & z_{1mk} \\ z_{21k} & - & z_{23k} & \cdots & z_{2mk} \\ z_{31k} & z_{32k} & - & \cdots & z_{3mk} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ z_{m1k} & z_{m2k} & z_{m3k} & \cdots & - \end{array} \right) \end{matrix}$$

Dealing with inter-region correlations (IRCs)

- Complexities of IRCs

- Some region pairs are unrelated, but others are correlated
- Correlation structure is intricate
- $0 \leq \rho \leq 0.5$
- Can we do a better job than GLMs or dichotomization?
 - Challenge: **How to characterize the complex structure?**



IRC: switching from GLM to LME

- IRC analysis through linear mixed-effects (LME) modeling

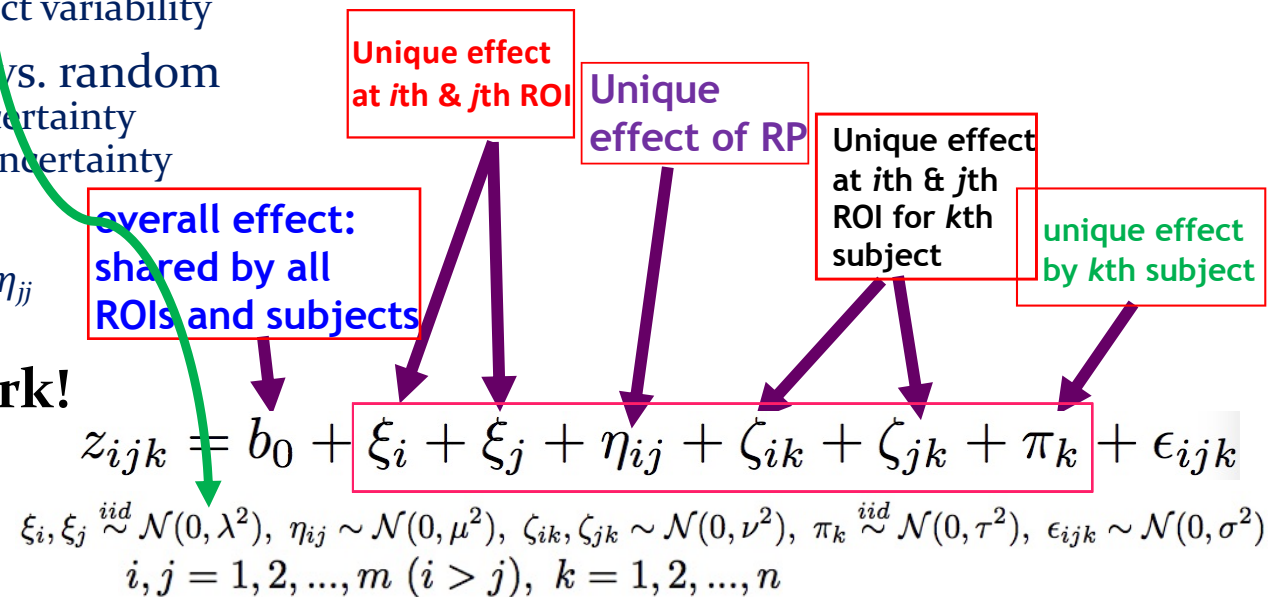
- One model integrates all ROIs: LME
- ROIs loosely constrained instead of being unrelated
 - Gaussian distribution: Is it far-fetched?
 - Similar to cross-subject variability
- Differentiation: fixed vs. random
 - Fixed: **epistemic** uncertainty
 - Random: **aleatoric** uncertainty

- Effects of interest

- region pair: $b_0 + \xi_i + \xi_j + \eta_{ij}$
- region: $0.5 * b_0 + \xi_i$

- LME wouldn't work!

Dead end!



IRC: one more jump from LME to **BML**

- **IRC analysis through Bayesian multilevel (BML) modeling**

- **One** model integrates all ROIs: BML (essentially same as LME)

- ROIs loosely constrained instead of being unrelated

- Gaussian distribution: Is it far-fetched?
 - Similar to cross-subject variability

- No differentiation: fixed vs. random

- All parameters: **aleatoric** uncertainty

- Effects of interest

- **region pair**: $b_0 + \xi_i + \xi_j + \eta_{ij}$
 - **region**: $0.5 * b_0 + \xi_i$

- LME plus **priors**

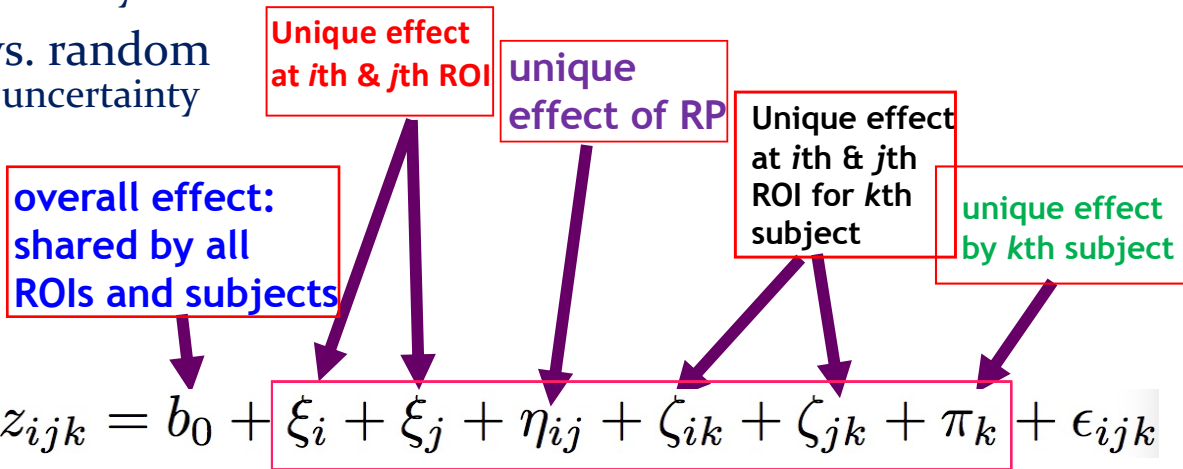
- **MCMC**

- Posterior distribution $z_{ijk} = b_0 + \xi_i + \xi_j + \eta_{ij} + \zeta_{ik} + \zeta_{jk} + \pi_k + \epsilon_{ijk}$

- **Ka-ching!**

$$\xi_i, \xi_j \stackrel{iid}{\sim} \mathcal{N}(0, \lambda^2), \eta_{ij} \sim \mathcal{N}(0, \mu^2), \zeta_{ik}, \zeta_{jk} \sim \mathcal{N}(0, \nu^2), \pi_k \stackrel{iid}{\sim} \mathcal{N}(0, \tau^2), \epsilon_{ijk} \sim \mathcal{N}(0, \sigma^2)$$

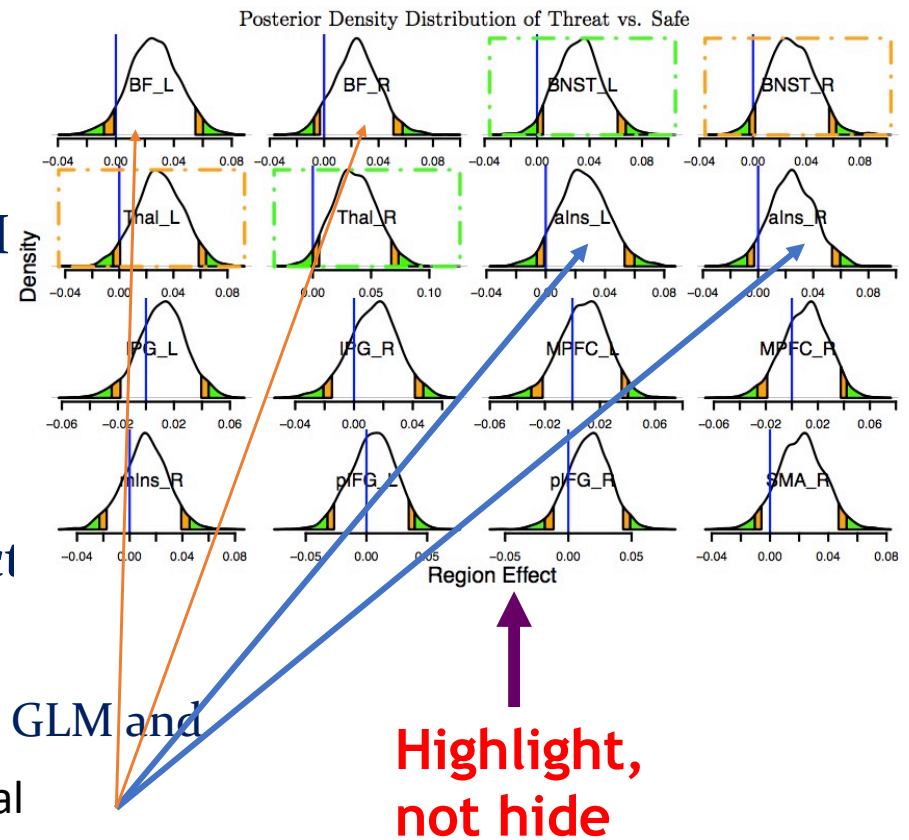
$$i, j = 1, 2, \dots, m \ (i > j), \ k = 1, 2, \dots, n$$



IRC – ROI effect from BML: full distributions

- ROI-based BML: 16 ROIs
- Full report with richer information: posterior distributions for each ROI
 - No dichotomization
 - Nothing hidden under sea level
 - $\rho = 0.483$
- 4 ROIs with strong evidence of effect compared to
 - Region effect inferences: unavailable from GLM and graph theory
 - Hubness?

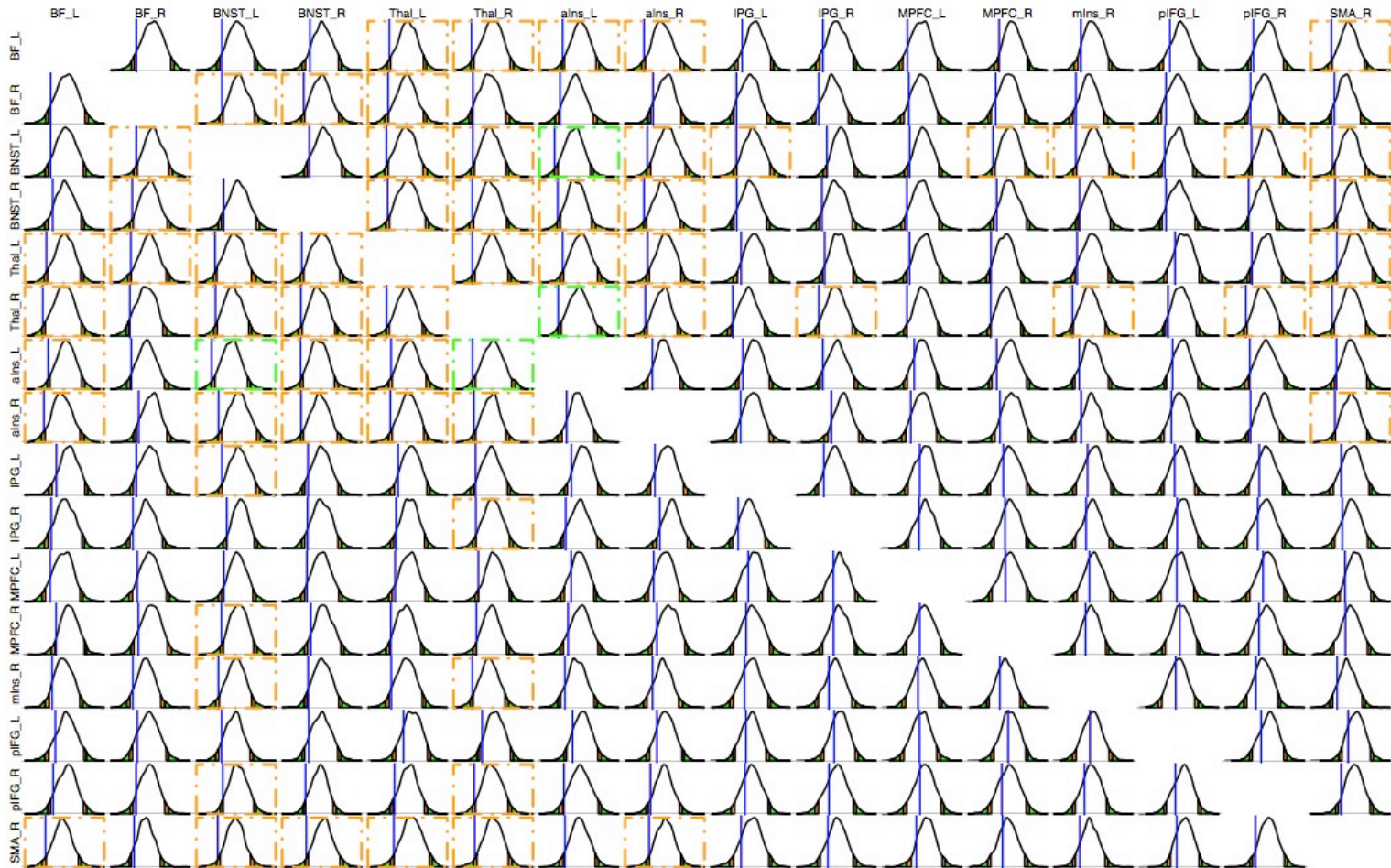
How about basal forebrain and Anterior Insula: L & R?



IRC – RP effect from BML: full distributions

120 RPs

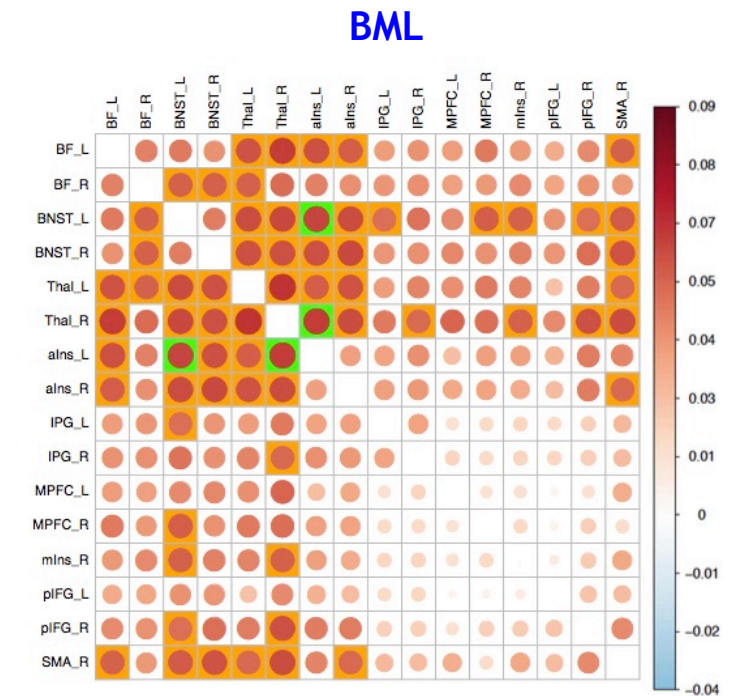
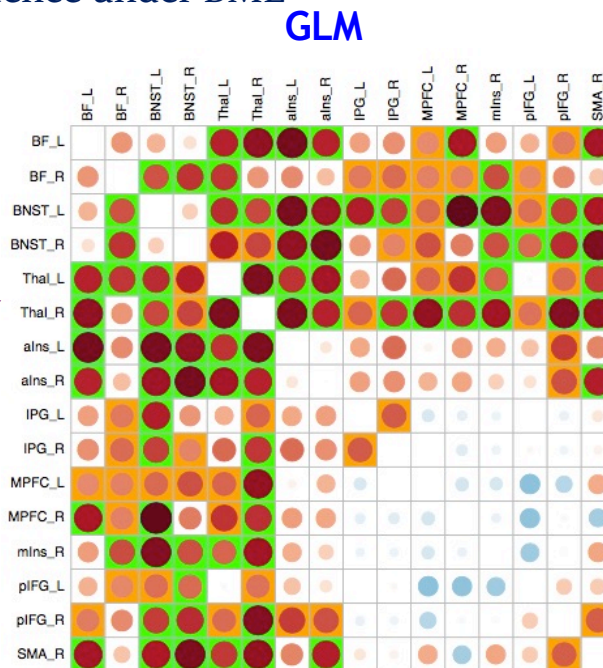
Highlight,
not hide



IRC- RP effect from BML

- ROI-based BML: 16 ROIs
- Full report for all region pairs (RPs)
- Comparisons with GLMs: **nothing hidden under sea level**
 - 63 RPs identified by GLMs with p of 0.05: **none survived** after correction with NBS via permutations
 - **33 RPs** with strong evidence under BML

Highlight,
not hide



BML: model validations

- ROI-based BML with IRD of 16

ROIs: cross-validation

- Leave-one-out information criterion (LOOIC)

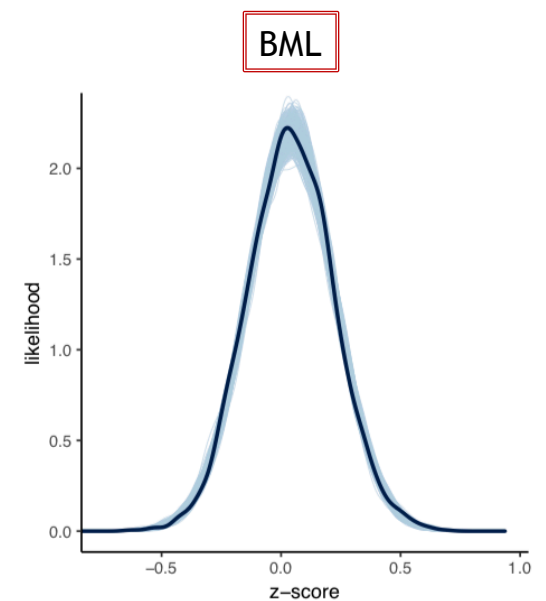
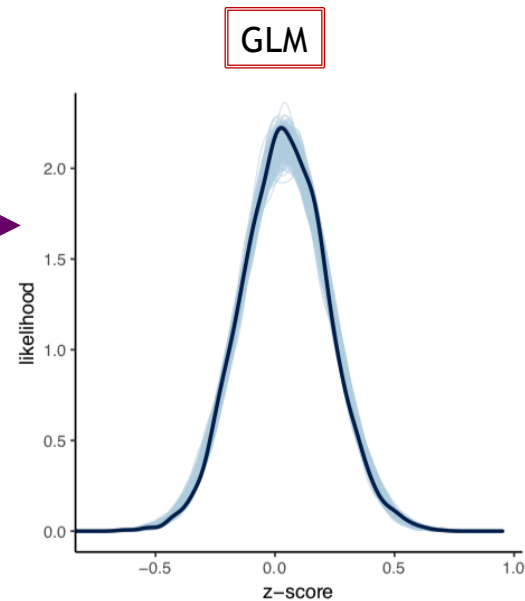
Cross-validation

Model	LOOIC	SE
GLM	-2808.31	101.65
BML0	-4543.77	102.97

- Posterior predictive checking →

• Effects of BML

- Regularizing ROIs: don't fully trust individual ROI data
- Sacrificing fit at each ROI; achieving better overall fit



Bayesian all the way

- **Should one correct for a duplicated study?**
- **How about all studies with statistical analyses**
- **Everyone is Bayesian**
 - Probabilistic nature
 - data; preprocessing, subjects, groups, sites, scanners, modeling approaches
 - Reproducibility
 - Most studies: similar; minority: outliers
 - [Applying a Gaussian prior](#)
- **Embracing, not fighting, multiplicity!**

Contrast

- **Mass Univariate Approach**

- Accurate with the current data, but poor for predictions
- Trust effect estimates (unbiased), but don't report them
- Doubt about statistical evidence, but selectively report it through filtering with colorbar and in table

- **BML**

- Compromise with the current data, but gain accuracy for predictions
- Pool effect estimate toward the center (biased), and directly show them through posterior distributions
- Statistical evidence shown without filtering

Summary

- **Issues with current correction for multiplicity**
- **Two toy examples**
 - NBA players
 - Kidney cancer
- **Application: region-based analysis (RBA)**
 - Program in **AFNI: RBA**
- **Other applications**
 - Matrix-based analysis (program in **AFNI: MBA**)
 - Region-based inter-subject correlation (ISC) analysis
 - Gray matter connectivity analysis
 - Others cases involving multiplicity

Keep Kidney Cancer in Mind!

- Kidney cancer distribution among counties

Highest rate



lowest rate



Calibration, regularization, information sharing, partial pooling, shrinkage

Acknowledgements

- **Paul-Christian Bürkner** (Aalto University, Finland)
- **Andrew Gelman** (Columbia University), **Stan Development Team, R Foundation**
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