If testing a 2-sided hypothesis, use a 2-sided test!

 \rightarrow for null hypothesis H_0 : μ = 0 , then:

$$\begin{array}{ll} \underline{1-\mathtt{sided}\ H_{\underline{a}}\mathtt{s}} & \underline{2-\mathtt{sided}\ H_{\underline{a}}\mathtt{s}} \\ H_{a}: \mu < 0 & H_{a}: \mu \neq 0 \\ H_{a}: \mu > 0 & H_{a}: \mu < 0 \text{ or } H_{a}: \mu > 0 \end{array}$$

If testing a 2-sided hypothesis, use a 2-sided test!

 \rightarrow for null hypothesis H_0 : μ = 0 , then:

1-sided H_a s

$$H_{a}$$
: $\mu < 0$
 H_{a} : $\mu > 0$

$$H_{\rm a}$$
: $\mu > 0$

either can be fine *if* prior info supports use

2-sided H_as

$$H_a$$
: $\mu \neq 0$

$$H_{\rm a}$$
: μ < 0 or $H_{\rm a}$: μ > 0

If testing a 2-sided hypothesis, use a 2-sided test!

 \rightarrow for null hypothesis $H_{\rm o}$: μ = 0 , then:

1-sided H_a s

$$H_a$$
: μ < 0

$$H_a$$
: $\mu > 0$

2-sided H_a s

$$H_a$$
: $\mu \neq 0$

$$H_{\rm a}$$
: μ < 0 or $H_{\rm a}$: μ > 0

logically identical statements, but doing a pair of 1-sided tests without multiple testing correction *necessarily* leads to FPR inflation

If testing a 2-sided hypothesis, use a 2-sided test!

Why FPR inflation if using a pair of 1-sided tests?

FPR
$$\alpha$$
 = area of tail(s)

2-sided

 $\alpha = 0.1$

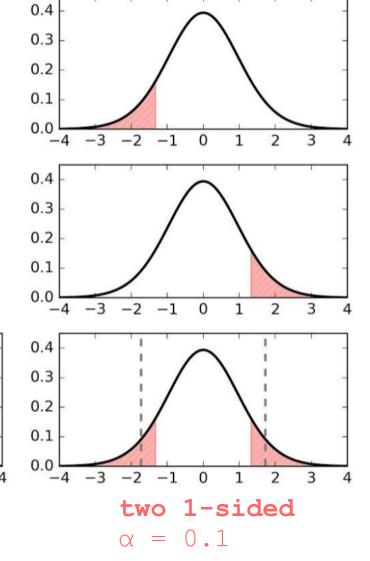
final: H_o rejection

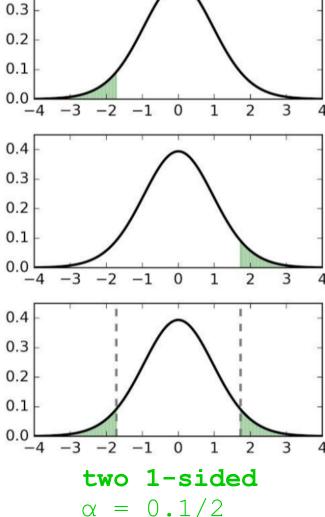
0.4

0.3

0.2

0.1





0.4

If testing a 2-sided hypothesis, use a 2-sided test!

Why FPR inflation if using a pair of 1-sided tests?

FPR
$$\alpha$$
 = area of tail(s)

Good

2-sided

 $\alpha = 0.1$

final: H_o rejection

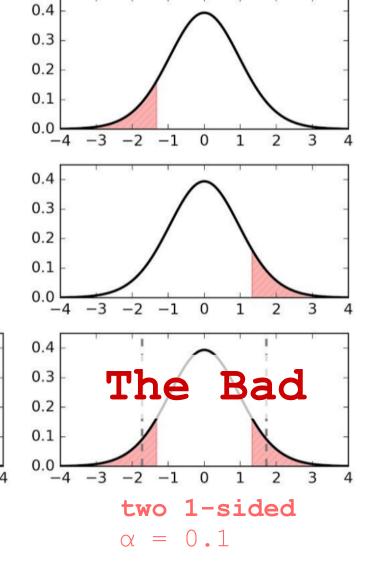
0.4

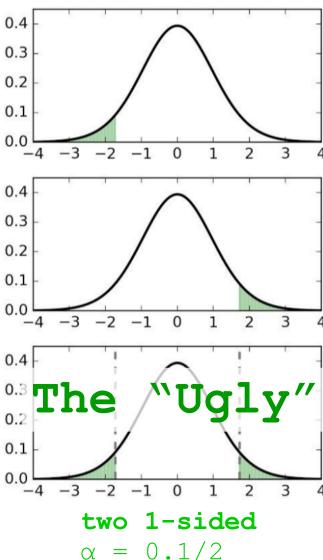
0.3

0.2

0.1

The /





correct

2-sided test = 0.001, $\alpha = 0.05$

<u>incorrect</u>

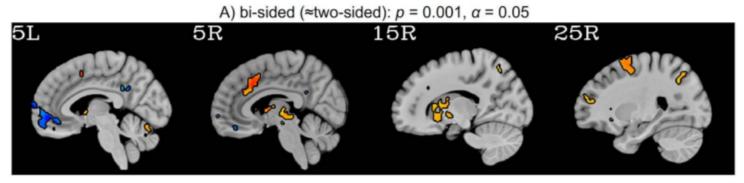
1-sided test**s** = 0.001, $\alpha = 0.05$

<u>incorrect</u>

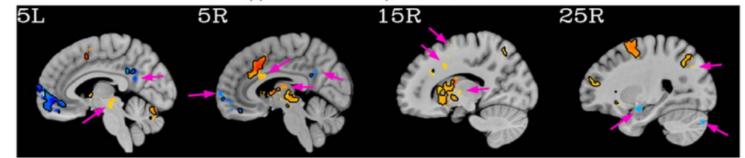
1-sided tests = 0.001/2 $\alpha = 0.05$

OK, but why??

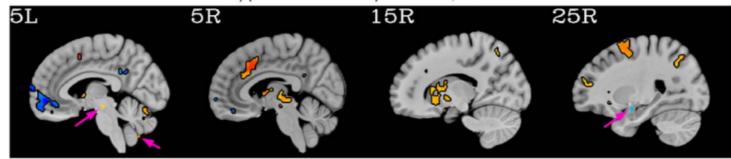
1-sided tests = 0.001/2 $\alpha = 0.05/2$



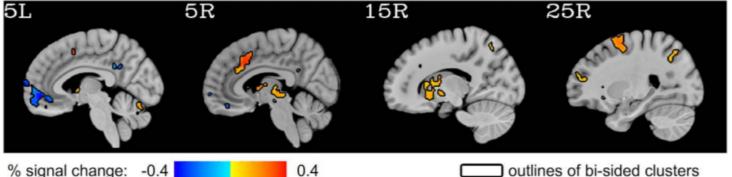
B) pair of one-sided: p = 0.001, $\alpha = 0.05$



C) pair of one-sided: p = 0.0005, $\alpha = 0.05$



D) pair of one-sided: p = 0.0005, $\alpha = 0.025$



% signal change: -0.4

large differences (cluster or

boundary) from bi-sided results

If testing a 2-sided hypothesis, use a 2-sided test!

Morals of the sidedness (or tail) tale:

- + A single, 1-sided test is fine if one has prior information and makes *a* 1-sided hypothesis.
- + For all other cases, use *a* 2-sided test.
- + A pair of 1-sided tests with FPR = α is equivalent to one 2-sided test with FPR = 2α , i.e., inflation without correction(s). Just use 2-sided!
- + Also, please report chosen test (+ rationale).

For more gruesome detail, see Chen et al. (2018) in bioRxiv (https://www.biorxiv.org/content/10.1101/328567v1) and in HBM (https://onlinelibrary.wiley.com/doi/full/10.1002/hbm.24399)