Improved characterisation of crossing fibres: spherical deconvolution combined with Tikhonov regularization

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Introduction: Diffusion-weighted MRI can be used in principle to extract the orientation of white matter fibres. Currently, the most common way of estimating the orientation of the fibres makes use of the diffusion tensor model. However, this model is unable to characterise voxels containing multiple fibre orientations [1]. The spherical deconvolution technique has recently been introduced to estimate the fibre orientation distribution (FOD) directly from the diffusion-weighted data [2]. An additional improvement has been proposed to obtain the optimal combination of response function (the convolution kernel) and low-pass filter [3]. However, conventional low-pass filtering reduces the effective angular resolution of the technique, making it more difficult to resolve fibre bundles if the angle between their orientations is too small. In this study, we present a two-pass Tikhonov regularisation method to significantly improve the angular resolution, using the fact that negative lobes in the reconstructed fibre orientation distribution (FOD) are unphysical.

Theory: The spherical deconvolution technique works by assuming an axially symmetric *response function* R(θ), corresponding to the diffusion-weighted signal attenuation that would be measured as a function of orientation for a coherently oriented fibre bundle aligned with the *z*-axis. The fibre orientation distribution (FOD), F(θ,ϕ), is then obtained by deconvolving this response function from S(θ,ϕ), the diffusion-weighted signal attenuation actually measured during the experiment [2]. The spherical convolution operation can be written (⊗ denotes the convolution operator):

$$S(\theta,\phi) = R(\theta) \otimes F(\theta,\phi)$$

This can be performed simply and efficiently using the spherical harmonic (SH) series [4]:

$$S^l_m = R^l \cdot F^l_m$$

where S^l_m and F^l_m denote the (*l*,*m*) SH coefficient of S(θ,ϕ) and F(θ,ϕ) respectively, and R^l is related to the (*l*,*m*=0) SH coefficient of R(θ) (all *m*≠0 coefficients are zero due to the assumed axial symmetry of the response function) [2]. The deconvolution operation can be performed by inverting the equation. However, as higher orders are included in the harmonic series, the operation becomes increasingly unstable due to the presence of noise. This can be overcome by introducing filtering, whereby the higher harmonic orders are attenuated by scaling the corresponding coefficients by a factor β^l, at the expense of angular resolution in the resulting FOD. These attenuation coefficients can be combined with the response function coefficients to give the so-called *effective* filtered response function, which can be optimised using a minimum entropy minimisation approach [3].

Due to the presence of noise and truncation artefacts, the FOD obtained using the optimised effective response function will contain some negative lobes, which are obviously unphysical. It is reasonable to assume that the FOD should be zero along these orientations. This information can be used to constrain the high angular frequency terms, without using an explicit low-pass filter. This type of constrained problem is commonly solved using Tikhonov regularisation methods, where the constraint is included as an additional term (weighted by a regularisation parameter λ) in a least squares minimisation problem [5].

Methods: Diffusion-weighted data were acquired from a healthy volunteer on a 1.5T Siemens Avanto system using a twice-refocused EPI sequence (b = 3000 s/mm^2, FOV = 235 × 235 mm, matrix size 112 × 112, slice thickness 3 mm, 37 contiguous slices, 60 directions, 6 b=0 images). The first pass of the algorithm uses the optimised effective filtered response function [3]. The second pass uses an estimate of the true response function *with no filtering*, obtained by averaging the DW signal in the 350 most anisotropic voxels in the white matter [2].

The following procedure was performed for each voxel. First, the SH decomposition of the optimally filtered FOD (labelled f_1) was estimated using the optimised effective filtered response function [3]. The amplitude of f_1 was then calculated along a set of 300 directions uniformly distributed over spherical coordinates. The orientations along which f_1 was negative were identified and used to produce the constraint matrix **L**; this matrix maps the SH decomposition of f_1 to its amplitude along the directions identified. The regularised FOD f_λ can then be estimated by performing the deconvolution using the response function estimated from the data *without* low-pass filtering, by formulating the problem as a Tikhonov regularisation with constraint matrix **L**, as follows:

$$f_\lambda = \text{argmin} \{ \| \mathbf{A}f - s \|^2 + \lambda^2 \| \mathbf{L}f \|^2 \}$$

where f is the vector of SH coefficients of the FOD, **A** is the matrix that maps f to the signal amplitudes along the orientations used in the acquisition scheme assuming the response function estimated from the data, s is the vector of signal amplitudes actually acquired, λ is the empirically determined regularisation parameter, and $\| x \|$ denotes the norm of vector x. This equation can be expressed as a simple linear least-squares problem, and easily solved using standard methods. In this study, all computations were performed using in-house software written in C++.

Results & Discussion: Figure 1 shows FODs over a 5×3 ROI located in the vicinity of the arcuate fasciculus, at the interface between two fibre bundles oriented at a relatively shallow angle. The improvement in the angular resolution brought about by the regularisation is evident in all voxels: the lobes for all fibre orientations are narrower and better defined. In addition, there are a number of voxels where the two fibre orientations could not be resolved clearly without the additional regularisation. The procedure also reduces the slight bias in the peak orientations that occurs when two peaks are close together. This bias is due to the interactions between their angular point spread functions, and can readily be observed in simple Cartesian space with two overlapping sinc or Gaussian functions. The improvement in angular resolution reduces the width of the effective angular point spread function, and thus reduces unwanted interference between the two peaks.

The constraint used in the regularisation is based on a reliable initial estimate of the FOD [2], and the physical reality of the problem. The proposed procedure should not therefore introduce unwanted artefacts into the reconstruction of the FOD. The value of the regularisation parameter λ used in this study was set empirically to the smallest value that would effectively remove spurious noisy peaks. Techniques for determining the optimal λ more objectively could be used, for example using the L-curve criterion [5].

Conclusion: We have described a method to significantly improve the angular resolution of the FOD obtained by spherical deconvolution, using the physical constraints of the problem as a priori information. With this method it is possible to resolve crossing fibres oriented at much smaller angles than was previously possible. This improved angular resolution is essential for reliable fibre-tracking through crossing fibres regions.

References: [1] Alexander DC *et al.* MRM 48:331-340 (2002). [2] Tournier JD *et al.* NeuroImage 23:1176-1185 (2004). [3] Tournier JD *et al.* Proc. ISMRM 13:384 (2005). [4] Healy DM *et al.* J. Mutlivar. Anal. 67:1-22 (1998). [5] Hansen PC. Numerical Algorithms 6:1-35 (1994).

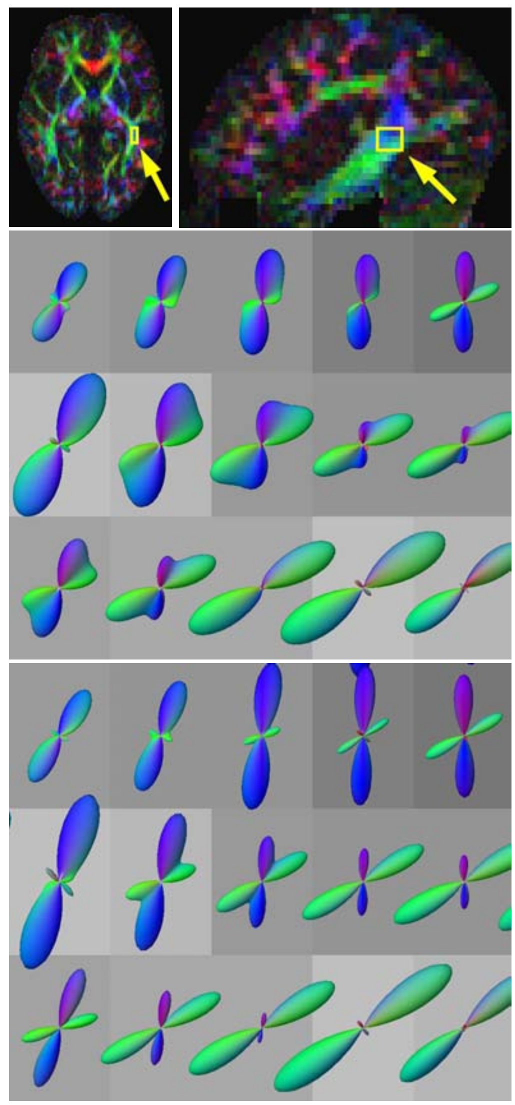

Figure 1: top: axial and sagittal color-coded eigenvector maps showing the location of the ROI, indicated by the arrow. Middle: FODs obtained with the optimised effective response function [3]. Bottom: FODs obtained using the proposed Tikhonov regularisation method. Results are projected sagittally in both cases.