

Spherical Harmonic Inductive Detection (SHID) Coils for Negative Feedback Control of Eddy Current Fields

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Introduction

Despite the best efforts of shielded gradient coil designers, flux leakage during gradient switching induces eddy currents in surrounding metal. The transient fields $B_z(t)$ then generated can have a spatial variation that is anything but a linear gradient. Working in spherical polar coordinates, these fields at some point $P(r, \theta, \phi)$ may be a sum of spherical harmonics $r^n P_n^m(\cos \theta) e^{im\phi} H_n^m(t)$ of order n and degree m , where P are the associated Legendre polynomials and $H(t)$ are functions of time. Typically, the lowest order fields, including $n = 0$, are greatest. To annul such fields one could tediously measure the various time dependences H for each harmonic and then arrange for the appropriate annulling transient currents to be passed through the shim coils. However, with any change of experiment or equipment, the process could well have to be repeated. We therefore research a different approach: measure each changing spherical harmonic field by electromagnetic induction with the aid of a SHID (Spherical Harmonic Inductive Detection) coil dedicated to detecting that harmonic alone, and then, in a negative feedback configuration, pass the temporal integral of the induced voltage (to some limiting, very-low frequency to avoid drift) back to the appropriate shim coil. Such "flux stabilisation" was a staple of early resistive magnet design for $n = 0, m = 0$; we aim to extend the principle to higher orders and degrees and report here on the successful design and fabrication of the necessary sensing coils.

Theory

The e.m.f. ξ induced in an arbitrary wire path l by a changing magnetic field \mathbf{B} is given by

$$\xi = -\frac{\partial}{\partial t} \oint \mathbf{A} \cdot d\mathbf{l}; \quad \mathbf{B} = \text{curl} \mathbf{A} \quad [1]$$

Our task then is to compute vector potential \mathbf{A} for each spherical harmonic field B_z and then determine a wire path that sets ξ to zero for all harmonics but the one of interest. Let the wire path be distributed in the magnet bore over the surface of a cylinder of radius a . Then in volume dV , $d\mathbf{l} = \mathbf{W} dV$ and vector winding density \mathbf{W} may be decomposed into axial z and azimuthal ψ components $W_z(z, \psi)$ and $W_\psi(z, \psi)$, with the azimuthal variations expressed as Fourier series in a familiar manner. Their Fourier coefficients are respectively $F_m(z)$ and $G_m(z)$. As the windings are confined to the surface, they must obey the continuity equation $\text{div} \mathbf{W} = 0$, and hence, as usual, if we know W_z we know W_ψ and vice versa.

Under the quasi-static approximation, \mathbf{B} is both $\text{curl} \mathbf{A}$ and the gradient of a scalar potential Ψ . Now, the individual Cartesian components of \mathbf{B} and \mathbf{A} are solutions to Laplace's equation, as is Ψ , and all can be written in terms of spherical harmonics. Relationships between these components can be found by equating the magnetic field derived from \mathbf{A} with the field from the scalar potential. When the algebra is complete, the harmonic component of the z -directed magnetic field

$$B_{z,n,m} = B_{a,n,m} T_{n,m} + B_{b,n,m} T_{n,m}' ; \quad T_{n,m} = r^n P_{n,m}(\cos(\theta)) \cos(m\phi); \quad T_{n,m}' = r^n P_{n,m}(\cos(\theta)) \sin(m\phi) \quad [2]$$

is related to the Cartesian components of the vector potential through $B_{a,n,m}$ and $B_{b,n,m}$ as:

$$A_{x,n,m} = \{B_{b,n,m} T_{n+1,m+1} - B_{a,n,m} T_{n+1,m+1}'\} A_{y,n,m} = \{B_{a,n,m} T_{n+1,m+1} + B_{b,n,m} T_{n+1,m+1}'\} A_{z,n,m} = \frac{(1-\delta_{m,0})}{(n+m+1)} \{ -B_{b,n,m} T_{n+1,m} + B_{a,n,m} T_{n+1,m}' \} \beta = \frac{1}{(n+m+1)(n+m+2)} \quad [3]$$

Integrating $\mathbf{A} \cdot \mathbf{W}$ azimuthally gives a non-zero result only when the degrees $m+1$ of A_x and A_y , and m' of \mathbf{W} are equal. In other words, a winding-density Fourier component that varies as $\cos(m'\psi)$ only responds to a field that varies as $\cos(m\psi)$ when $m' = m$. As we now know the required azimuthal variation of \mathbf{W} to detect a field of degree m , it remains to determine the required axial variation.

Integrating axially the results of the azimuthal integration, we arrive at an expression (essentially Φ_T the winding flux linkage) whose temporal derivative gives the voltage induced in the windings. Turning to a matrix representation, let discrete axial winding positions on the cylinder be given by $z = q\Delta z$, where q runs from 1 to q_{max} . The flux linkage for sinusoidal variations is then:

$$\Phi_T = -\pi a \Delta z \sum_{n=m}^{\infty} \frac{B_{a,n,m}}{(n+m+1)} \left[\frac{(1+\delta_{m,0})}{(n+m+2)} \sum_q f_q^{(n+1)} P_{n+1,m+1}(\cos(\alpha_q)) G_{m,q} + (1-\delta_{m,0}) \sum_q f_q^{(n+1)} P_{n+1,m}(\cos(\alpha_q)) F_{m,q} \right] \quad [4]$$

where (f_q, α_q) represent the spherical coordinates of $q\Delta z$, and the winding Fourier coefficients of degree m are $F_m(q\Delta z)$ and $G_m(q\Delta z)$.

Eqn [4] can be written in a matrix form $\Phi_m = \mathbf{S}_m \cdot \mathbf{W}_m$, where Φ_m is a column vector of the flux contributions from the various orders n of the magnetic field of degree m , \mathbf{W}_m is a vector comprising the Fourier components of the wire density and \mathbf{S}_m is a rectangular matrix depending on the coefficients $B_{a,n,m}$. The sum of the elements of Φ_m equals the total flux, Φ_T , from eqn [4]. The matrix equation is extended with sinusoidal variations and constraints $\text{div} \mathbf{W} = 0$ and no wire at cylinder ends (1). To design a SHID coil, all elements of Φ_m , except that desired, are set to zero and the elements of \mathbf{W}_m are determined using the pseudo-inverse of \mathbf{S}_m (1).

Simulation:

To determine the effects of the discretisation and interconnect process when using the stream function of the winding density to reveal the wire path, a simulation program was written. The simulator integrated the vector potentials of Eq. [3] along the SHID wire path. When the Fourier coefficients of the continuous wire distribution were placed in Eq. [4] it was found that they would reject unwanted harmonics by 16 orders of magnitude. The simulator revealed that the rejection of unwanted harmonics depends on the number of contours taken from the stream function of \mathbf{W}_m . Not surprisingly, the more contours, the better the approximation of the continuous function and the greater the rejection. For example the sparsely wound $n = 2, m = 1$ SHID coil with layout shown in the figure has a response to an $n = 4, m = 1$ harmonic that is 3800 times less than its response to the 2,1 harmonic while responses to all other harmonics are reduced even further.

Discussion

The question arises of what weights to assign the $B_{a,n,m}$ coefficients in Eq. [4], when the matrix \mathbf{S}_m is constructed, in the light of their r^n dependence and computational rounding errors. The coefficients describe a fictitious magnetic field against which the SHID coil is designed. Ideally the flux response of a SHID coil should be proportional to one of these terms and independent of all the others making any non-zero choice valid. In practice the designs are slightly imperfect. If a particular $B_{a,n,m}$ is chosen proportionally much smaller than the other coefficients, the rejection ratio for that harmonic could be degraded. Thus some scheme should be employed to put the $B_{a,n,m}$'s on a fair playing field. Choosing the coefficients to represent a magnetic field where each harmonic component has the same amplitude on the spherical surface of the region of interest produced good results. Finally, testing of the SHID coils with a standard z gradient coil in a model system is underway and proceeds well. It is planned to use a 20 cm diameter aluminium pipe (not necessarily coaxial) to generate eddy current fields.

Reference

1. D.I. Hoult and R. Deslauriers. *Journal of Magnetic Resonance A*, **108** 9-20, 1994.

