

Chemical Exchange in Fully-Balanced Steady-State Free Precession

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INTRODUCTION: With recent improvements in scanner hardware, specifically gradient performance, allowing fully-balanced steady-state free precession (SSFP) images to be acquired with repetition times (TR) of less than 5ms, SSFP has become an attractive imaging option - affording rapid acquisition times with high signal-to-noise ratio. Short TR values are vital in SSFP in order to minimize the appearance of off-resonance banding artifacts. Although several biological tissues, such as white and grey matter, have been shown to exhibit multi-component relaxation¹, the generally accepted SSFP signal equation²,

$$SSFP = \rho(1 - e^{-TR/T_1})e^{-TR/T_2} \sin\alpha / (1 - e^{-TR/T_1}e^{-TR/T_2} - (e^{-TR/T_1} - e^{-TR/T_2})\cos\alpha), \quad [1]$$

assumes only a single T_1 and T_2 relaxation time and makes no account of chemical exchange within the imaged system. These effects are important, both for modeling the measured SSFP signal, as well as to investigate their influence on the DESPOT2 (Driven Equilibrium Single Pulse Observation of T_2) T_2 mapping approach³, based on SSFP. In this work we include 2-site chemical exchange into the general SSFP magnetization expressions and use simulations to determine the effect on the measured signal from each species.

THEORY: Expressing the Bloch-McConnell⁴ expressions describing the 2-component system (a and b) as

$$\frac{dM}{dt} = AM - C, \text{ with,}$$

$$A = \begin{bmatrix} -1/T_{2,a} - k_{ab} & k_{ba} & \Delta\omega_a & 0 & 0 & 0 \\ k_{ab} & -1/T_{2,b} - k_{ba} & 0 & \Delta\omega_b & 0 & 0 \\ -\Delta\omega_a & 0 & -1/T_{2,a} - k_{ab} & k_{ba} & 0 & 0 \\ 0 & -\Delta\omega_b & k_{ab} & -1/T_{2,b} - k_{ba} & 0 & 0 \\ 0 & 0 & 0 & 0 & -1/T_{1,a} - k_{ab} & k_{ba} \\ 0 & 0 & 0 & 0 & k_{ab} & -1/T_{1,b} - k_{ba} \end{bmatrix}, \quad M = \begin{bmatrix} M_{a,x} \\ M_{b,x} \\ M_{a,y} \\ M_{b,y} \\ M_{a,z} \\ M_{b,z} \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ f_a M_o / T_{1,a} \\ f_b M_o / T_{1,b} \end{bmatrix}$$

where k_{ab} and k_{ba} are exchange rates between a and b , $\Delta\omega$ denotes off-resonance, and f is the volume fraction, we find the discrete-time solution,

$$M^N = (e^{A \cdot TR} \times R(\alpha))M^{N-1} + (e^{A \cdot TR} - I)A^{-1}C. \quad [2]$$

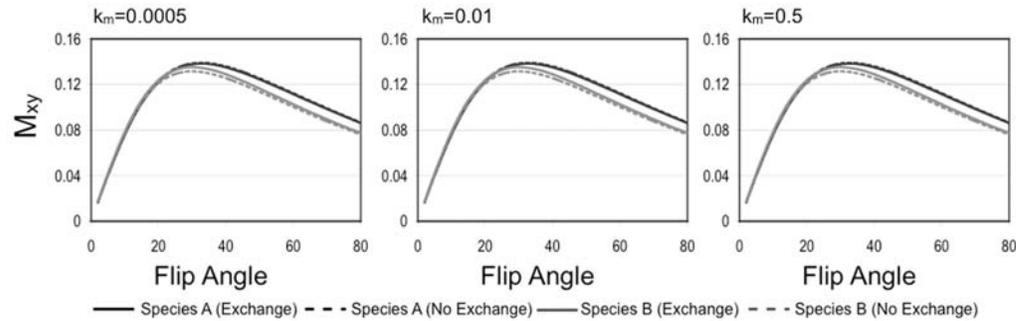
where $R(\alpha)$ is the rotation matrix acting on M_a and M_b , I is the 6x6 identity matrix, and M^N and M^{N-1} are the magnetization vectors immediately preceding two successive RF pulses. Imposing the steady-state condition ($M_{SS} = M^N = M^{N-1}$),

yields, $M_{SS} = (I - R(\alpha)e^{A \cdot TR})^{-1} \times (e^{A \cdot TR} - I)A^{-1}C$ with the measured signal

$$SSFP_2 = M_{SS,x} + iM_{SS,y}. \quad [3]$$

METHODS: The effect of exchange on the steady-state signal was investigated by comparing signal intensity vs. α curves for each species calculated using Eqns [1] and [2] at $\alpha = 1^\circ$ to 80° , over a range of k_m ($k_m = f_a k_{ab} + f_b k_{ba}$) values (0.0005 to 1ms^{-1}), with the following assumed values: $T_{1,a}=350\text{ms}$, $T_{1,b}=1200\text{ms}$, $T_{2,a}=30\text{ms}$, $T_{2,b}=120\text{ms}$, $f_a=0.2$, $TR=5\text{ms}$ and $\Delta\omega = 0^\circ$.

RESULTS / DISCUSSION: Example SSFP vs. α curves are shown in Fig. 1, corresponding to slow ($k_m=0.0005$), intermediate ($k_m=0.01$) and fast ($k_m=0.5$) exchange.



For all k_m values, close agreement is observed between the signal curves of each species with and without exchange, suggesting that at the TR value used (5ms), typical of most SSFP experiments, the SSFP2 signal may be modelled using a no-exchange approximation,

$$SSFP_2 = M_o \left[\frac{f_a(1 - E_{1,a})E_{2,a} \sin\alpha}{1 - E_{1,a}E_{2,a} - (E_{1,a} - E_{2,a})\cos\alpha} + \frac{f_b(1 - E_{1,b})E_{2,b} \sin\alpha}{1 - E_{1,b}E_{2,b} - (E_{1,b} - E_{2,b})\cos\alpha} \right] \quad [4]$$

where $E_{1,i} = e^{-TR/T_{1,i}}$, $E_{2,i} = e^{-TR/T_{2,i}}$. Eqn. [4] suggests that SSFP may provide a new means of observing and quantifying multi-component relaxation. Further, as DESPOT2, involves acquisition of SSFP data with constant TR , the relative contribution of each species to the signal is constant for all data, in contrast with spin-echo.

REFERENCES: [1] Whittall K. *et al.* MRM 37:34-43 (1997), [2] Hinshaw WS J Appl Phys 47:3709-3712 (1976), [3] Deoni SCL *et al.* MRM 46:515-526 (2003), [4] McConnell HM J Chem Phys 28:430-431 (1958), [5] Does M *et al.* MRI 16:1033-1041 (1998)