

# Phase-Compensated Spin-Echo Sequence Using Hyperbolic Secant Pulses for Both Excitation and Refocusing

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**Introduction:** Frequency-modulated (FM) pulses, which includes the class of adiabatic full-passage (AFP) pulses, have been widely used for inversion in NMR due to their immunity to RF field inhomogeneity and ability to invert spins uniformly over broad bandwidths with relatively low peak RF power. However, when used for slice-selective refocusing, FM pulses have the disadvantage that the phase of the transverse magnetization varies in a non-linear manner across the slice. As one solution to this problem, a double spin-echo sequence using a pair of identical AFP pulses can be used to compensate the non-linear phase produced by the individual pulses (1). An alternative approach accomplishes refocusing with a single AFP pulse (specifically a chirp pulse), following excitation with a chirp pulse (2, 3). With the latter method, the quadratic phase profile can be compensated by appropriately setting the slice-selection gradient amplitude or the pulse frequency sweep velocity used with the individual pulses. A similar method was applied for the well-known hyperbolic secant (HS) pulse, whereby phase compensation is achieved by changing a truncation factor of the HS pulses to control the sweep rate (4). In the present work, the phase profiles are analytically described when HS pulses are used for both excitation and refocusing. Based upon this analysis, the exact condition to achieve phase compensation is obtained. In addition, multi-slice spin-echo imaging using HS pulses is demonstrated in Bloch simulations and a phantom experiment. Experimental performance obtained with HS versus sinc pulses are also compared.

**Theory:** In the FM rotating frame, the HS pulse can be described by AM and FM functions:  $\omega_1(t) = \omega^{max} \text{sech}(\beta\tau)$ ,  $\omega_2(t) = A \tanh(\beta\tau)$ , where  $\tau$  is normalized time ( $=2t/T_p - 1$ ) for  $t$  in the range  $[0, T_p]$ ,  $A$  is the amplitude of frequency sweep (rad/s), and  $\beta$  is a truncation factor (usually,  $\text{sech}(\beta) = 0.01$ ). When an HS pulse of length  $T_p$  is applied, spins with offset frequency  $\Omega$  are assumed to be *instantaneously* rotated at time  $t_\Omega = T_p(4\beta)^{-1} \ln((A+\Omega)/(A-\Omega)) + T_p/2$ . Below, the  $\beta$  and  $T_p$  parameters of the excitation and refocusing pulse are indicated by a subscript of 1 and 2, respectively. By assuming an isochromat freely evolves after resonance is achieved during the pulse, the phase accrued at the end of HS excitation can be written as  $\phi(\Omega, T_{p,1}) = \phi_{HS}(t_{\Omega,1}) + \Omega T_{p,1} - t_{\Omega,1} + \pi/2$ , where  $\phi_{HS}$  is given by the integration of the FM function. And, assuming a delay time  $\Delta$ , the phase accumulated by the end of HS refocusing becomes  $\phi(\Omega, T_{p,1} + \Delta + T_{p,2}) = -\Omega \phi(\Omega, T_{p,1}) + \Delta + t_{\Omega,2} + 2\phi_{HS}(t_{\Omega,1}) + \Omega T_{p,2} - t_{\Omega,2}$ , where  $2\phi_{HS}$ , not  $\phi_{HS}$ , is added because the isochromat is reflected about the axis of  $\phi_{HS}$  in the same plane. Therefore, following refocusing ( $t'=0$ ), the total phase accrued at time  $t'$  is given by

$$\phi(\Omega, t') = A \left\{ \frac{T_{p,2}}{\beta_2} \ln \left( \frac{A \cdot \text{sech}(\beta_2)}{\sqrt{A^2 - \Omega^2}} \right) - \frac{T_{p,1}}{2\beta_1} \ln \left( \frac{A \cdot \text{sech}(\beta_1)}{\sqrt{A^2 - \Omega^2}} \right) \right\} - \frac{\Omega}{2} \left( \frac{T_{p,2}}{\beta_2} - \frac{T_{p,1}}{2\beta_1} \right) \ln \left( \frac{A + \Omega}{A - \Omega} \right) + \Omega \left( t' - \Delta - \frac{T_{p,1}}{2} \right) - \frac{\pi}{2} \quad [1]$$

From Eq.[1], if  $\beta_1 = \beta_2$ , the first and second terms that are non-linear in terms of  $\Omega$  disappear when  $T_{p,1} = 2T_{p,2}$ . This is the exact condition for phase compensation using HS pulses for both excitation and refocusing, which is in agreement with the result from the chirp pulse (3). On the other hand, if  $T_{p,1} = T_{p,2}$ , in the condition of  $\beta_1 = 0.5\beta_2$  the second non-linear term disappears, but the first term is not cancelled out completely because  $\text{sech}(\beta_1) \neq \text{sech}(\beta_2)$ . However, in this case, the non-linear phase can be regarded to be almost compensated to a good approximation, which explains the previous work using HS pulses (4).

**Methods:** To simulate the phase compensation, HS pulses with  $T_{p,1} = 2$ ms for excitation and with  $T_{p,2} = 1$ ms for refocusing were used having the same  $A/2\pi$  of 7.5 kHz. The delay time  $\Delta$  between excitation and refocusing was assumed to be 5 ms. The slice profile was also obtained by the Fourier transform of the echo signal. Experiments were performed at 4 Tesla using a TEM head resonator and a homogeneous cylindrical phantom (length = 18cm, diameter = 9cm) containing 0.45% NaCl solution and trace amounts of Gd-DTPA. HS pulses with  $T_{p,1} = 8$  ms and  $T_{p,2} = 4$  ms, with the same  $A/2\pi$  of 3.75 kHz, were used for excitation and refocusing, respectively, in a 2D spin-echo imaging sequence. For comparison, the experiment was repeated using sinc pulses with  $T_p = 4$  ms. In both cases, TE = 22 ms, TR = 2 s, THK = 5 mm, FOV = 30x30 cm<sup>2</sup>, and matrix size = 256x128.

**Results:** The result of the simulation (Fig. 1) shows that a spin-echo sequence using HS pulses provides an excellent slice profile and complete rephasing occurs for  $T_{p,1} = 2T_{p,2}$  in the pulse bandwidth ( $2A$ ). The experimental image of the phantom obtained with HS pulses (Fig. 2) is better quality than that obtained with sinc pulses (Fig. 3). With the HS pulses, SNR improved because the HS refocusing pulse functioned adiabatically and thus was insensitive to RF inhomogeneity. The maximum SNR increase was ~25% near the periphery of the phantom where the RF field was weakest.

**Conclusion:** In a spin-echo sequence using HS pulses, the phase compensation occurs for  $T_{p,1} = 2T_{p,2}$ . Unlike the chirp pulse, phase compensation cannot be achieved by controlling gradients because the HS pulse uses a non-linear frequency sweep. Although the HS pulse for excitation is sub-adiabatic, the full adiabaticity of the HS refocusing pulse offers a desirable degree of tolerance to RF inhomogeneity. Furthermore, this sequence produces an excellent slice profile because HS pulses yield nearly identical, highly uniform profiles when used for both excitation and refocusing. Finally, since it uses a single AFP for refocusing, it requires less TE and power deposition than other slice-selective spin-echo sequences using multiple AFP pulses for phase compensation.

**References:** (1) S. Conolly, *et. al*, *JMR* 78, 440-458 (1988) (2) D. Kunz, *MRM* 4, 129-136 (1987) (3) J-M Böhlen, *et. al*, *JMR* 84, 191-197 (1989) (4) K. Shmueli, *et. al*, *ISMRM* (2003)

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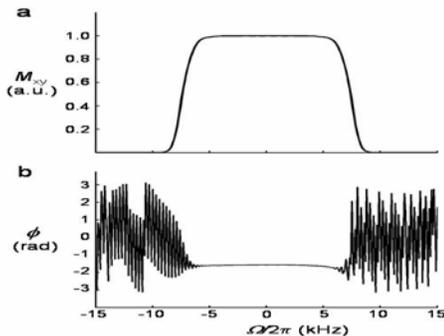


Fig.1 (a) Slice and (b) phase profiles obtained from a spin-echo sequence using HS pulses for excitation and refocusing ( $T_{p,1} = 2$ ms,  $T_{p,2} = 1$ ms,  $A/2\pi = 7.5$ kHz).

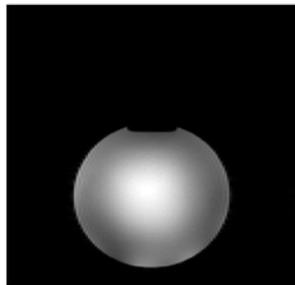


Fig.2 A phantom image obtained from 2D spin-echo imaging using HS pulses.

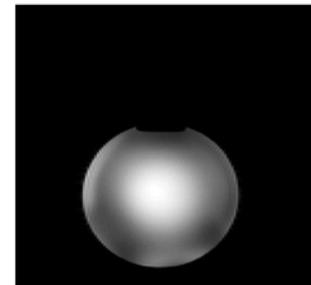


Fig.3 A phantom image obtained from 2D spin-echo imaging using HS pulses.