Fast Toeplitz Based Iterative SENSE Reconstruction

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Introduction

Reconstruction of images from sensitivity encoded imaging (SENSE) is computationally demanding when non-Cartesian k-space sampling is used for the data acquisition. Most of the reconstruction time is accounted for by the multiple matrix-vector multiplications required in conjugate gradient (CG) iterations. Several fast iterative reconstruction methods were introduced to reduce the reconstruction time using non-uniform FFT (NUFFT) [1] or linear convolution with circulant approximation [2,3]. The NUFFT based method can correct for the off-resonance, but it's slower than the other fast methods due to extra operations in temporal segmentation. The linear convolution methods are faster, but they do not correct for the off-resonance, thus the methods are more susceptible to field inhomogeneity when they are used in functional imaging. In this work, we present a new fast iterative reconstruction method for SENSE, which is significantly faster than the NUFFT based method, while providing off-resonance correction with high accuracy. A simulation study is performed to compare the reconstruction speed and the accuracy with the NUFFT based method.

Theory

In the presence of field inhomogeneity and/or relaxation, a reasonable discrete linear signal model for SENSE data acquired using *P* array coils is

$$Y_p = AS_p X \, , \; p = 1,\ldots,P \; ,$$

where Y_p is the data from *p*'th coil; A is the system matrix with the i,j'th element $[A]_{ij} = \exp(-i2\pi\vec{k}(t_i)\cdot\vec{r}_j)\exp(-iz_j t_i)$;

$z(\vec{r}_j) = \omega(\vec{r}_j) - iR_2^*(\vec{r}_j)$ is the rate map that contains off-resonance and relaxation; S_p is sensitivity map of the *p*'th coil; and *X* is the unknown image to be reconstructed.

The iterative reconstruction is performing by minimizing the regularized cost function using CG

$$\Psi(X) = \sum_{p=1}^{P}\left\|Y_p - AS_p X\right\|^2 + \beta\cdot R(X) \, ,$$

where *R(X)* is a spatial regularization function and β is the regularization parameter. Here, the computational bottleneck is the calculation of $\sum_{p=1}^{p=P} S'_p A' A S'_p X^{(n)}$ at each CG iteration, where the multiplication of $A'A$ to a vector is the most time consuming operation. Without field map (and relaxation map), $A'A$ would be Toeplitz, and the multiplications to a vector can be done efficiently using FFT [2-4]. However, $A'A$ is not Toeplitz when off-resonance is present in the system model. In this case, we can use a Toeplitz based approximations to $A'A$ [5] using a principle similar to time-segmentation [6]. In particular, $A'A \approx \sum_{l=1}^{L} D'_l T_l D_l$,where each T_l is a Toeplitz matrix and each D_l is a simple diagonal matrix. So multiplying $A'A$ to a vector requires only *LP* pairs of FFTs with no interpolations after the first iteration, which is why the proposed method is faster than the NUFFT based method with field map correction.

Methods

An NUFFT based iterative reconstruction and the proposed method were implemented using MATLAB and C. The data was generated without noise using exact DTFT. A two-times-undersampled (R=2) spiral trajectory was used with 1890 data samples, sampling time = 4us, FOV=22cm, TE=30ms, matrix size = 64X64, and number of coils P = 4. R2* decay was not included in the system for simplicity. Figure 1 (a), (b), and (c-f) show the reference image, field map, and the sensitivity maps. For the NUFFT based method and proposed method, the number of time segments was 6 and 8 respectively. The regularization parameter β was set to provide full width half maximum of point spread function = 1.32, and the number of iterations was 15.

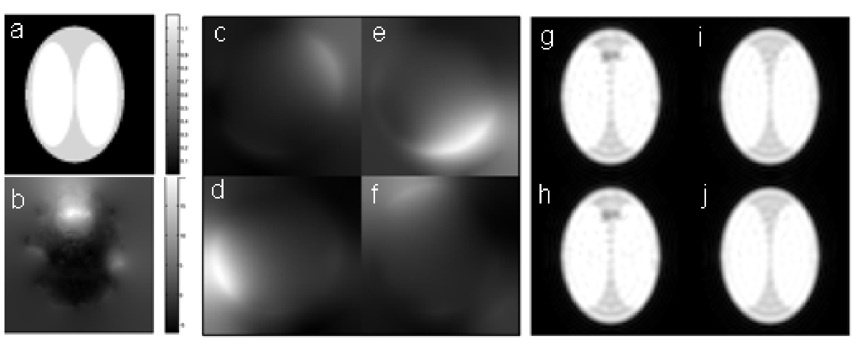

Figure 1. (a) True image X (b) field map (c-f) sensitivity maps (g) NUFFT based recon w/o field map correction (NRMSE = 87%) (h) proposed method w/o field map correction (NRMSE = 87%) (i) NUFFT based recon with field map correction (NRMSE = 7.4%) (j) proposed method with field map correction (NRMSE = 7.4%)

Results and discussion

For a given number of iterations, the NUFFT based method and the proposed method provided substantial field map correction, and indistinguishable image quality. However, the proposed method required only 6.6 cpu seconds on a 2.0 GHz Xeon, whereas the NUFFT based method required 13.8sec. The acceleration of the proposed method would be more dramatic using dedicated FFT hardware.

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References

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