Predicting SNR gains from constructive averaging of proton spectra: theory and practice

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Introduction

Destructive interference from the phase fluctuations caused by motion during 1H MRS STEAM and PRESS acquisitions can offset the improved signal-to-noise ratio (SNR) provided by averaging for weak metabolites such as creatine and choline. One solution, *constructive averaging,* is to apply the sequence with incomplete water-suppression, and phase each signal based on the water signal [1] or other peaks [2], prior to averaging. A second solution is to incorporate double-gating [3], or a navigator acquisition focused at the motion source, to reject MRS signals whose phases vary due to motion [4]. The SNR gain that results from constructive averaging can be manifold over that of conventional averaging in the presence of motion. However, the gain is highly variable and to date, unpredictable. Here we present an analytic theory that predicts the SNR advantage of constructively averaging spectral acquisitions subject to extraneous phase fluctuations. We show that the SNR improvement follows a simple analytical function of the standard deviation (SD) of the phase variation. The theory is validated using *in vivo* 1H spectra from the human heart and leg muscles.

Theory

Consider an MRS peak as a vector in the complex plane. Extraneous motion causes this vector to rotate. Assume that the random motion induces phases in all acquisitions that are independent and identically distributed (IID) uniformly in the interval $[-\theta_0, \theta_0]$. The interval $2\theta_0$ is related to the SD, σ_θ,of the phase by $2\theta_0 = \sqrt{12}\,\sigma_\theta$. With regular averaging, the constructive component is the average of the projection of each of these vectors on the x-axis, or: $S_{reg} = (1/NEX)\Sigma c_i$ where $c_i=\cos(\theta_i)$, θ_i is the phase of the i^{th} vector, and NEX is the number of acquisition. The random variables$\{c_i\}_{i=1..N}$ can be shown to be IID with a probability density function (PDF) of the form:

$f_C(c) = (1/\theta_0)[1/\sqrt{(1-c^2)}]$, with $c \in [\cos(\theta_0), 1]$. The mean and variance of c are: $\mu_c=\mathrm{sinc}(\theta_0/\pi)$ and $\sigma_c^2=0.5+0.5\mathrm{sinc}(2\theta_0/\pi)-\mathrm{sinc}^2(\theta_0/\pi)$ with $\mathrm{sinc}(x)=\sin(\pi x)/\pi x$. The mean averaged signal is $\mu_S=\mathrm{sinc}(\theta_0/\pi)$ with variance $\sigma_S^2=(1/N)[0.5+0.5\mathrm{sinc}(2\theta_0/\pi)-\mathrm{sinc}^2(\theta_0/\pi)]$. Dephasing thus reduces the signal by a factor $\mathrm{sinc}(\theta_0/\pi)$.

Now consider the constructively-averaged spectrum. Assume the spectrum has normally distributed IID complex components with the same variances; $x\sim N(\mu_x, \sigma)$ and $y\sim N(\mu_y, \sigma)$. The phase $\omega = \tan^{-1}(y/x)$ is given by the Rician distribution [5]. For SNR$\geq$3, ω is well approximated by a Gaussian PDF with mean α and variance $1/\mathrm{SNR}^2$, where $\alpha = \tan^{-1}(\mu_y/\mu_x)$, $\mathrm{SNR} = a/\sigma$, $a = (\mu_x^{\,2}+\mu_y^{\,2})^{1/2}$. The average phase of M independent samples yields a normal phase $\phi\sim N(\alpha, \sigma_\phi)$, with $\sigma_\phi=\sigma_\omega/\sqrt{M}$. The constructively-averaged signal is $S_{con} = (1/NEX)\Sigma c_i$ with $c_i=\cos(\phi_i-\alpha)$. It can be shown that S_{con} has a mean and variance of $\exp\{-1/(2M.\mathrm{SNR}^2)\}$ and $\{0.5+0.5\exp(-2/(M.\mathrm{SNR}^2))-\exp(-1/(M.\mathrm{SNR}^2))\}$, respectively. For a sufficiently large M and a good SNR, S_{con} is deterministic with a value of ~1. Since the noise distribution is unaffected by phasing, the SNR gain is the same as the ratio of signals, S_{con}/S_{reg}. Therefore, the expected SNR improvement from constructive averaging is also $\sim 1/\mathrm{sinc}(\theta_0/\pi)$.

Experiments

Sets of 64 and 128 (NEX) un-averaged STEAM and PRESS 1H MRS (TE = 10-15 ms) were collected on a GE 1.5 T Signa scanner, from the legs or hearts of 12 healthy volunteers using either an extremity 1H MRI coil, or a 13-cm surface receiver coil. Subjects were positioned prone on the surface coil for heart studies, and voxels prescribed from scout MRI, after auto-shimming. The average phase of $M=\sqrt{NEX}$ points centered on the lipid peak was used to phase all acquisitions in all data sets. SNR with constructive and conventional averaging were measured to quantify S_{reg}/S_{con},: the reciprocal of this is the SNR gain from constructive averaging. Fig. 1 shows the analytical sinc-function by which the SNR is reduced by motion, and 95% confidence interval. The measured signal loss with regular averaging vs constructive averaging as a function of θ_0 for the phase variation in the lipid peaks, is overlaid. Data are in excellent agreement with the theoretical curve. The SNR gain from constructive averaging was thus ~0% to 700%.

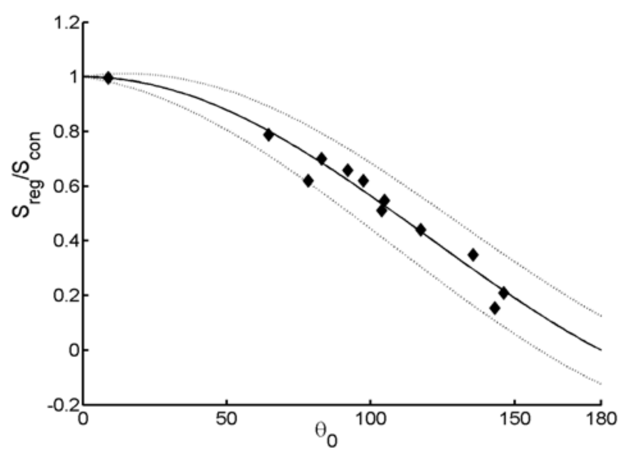

Fig. 1 *Predicted SNR reduction with regular averaging (solid: 95% confidence intervals, dotted), and measured 1H MRS data (points)*

Conclusion

The SNR advantage of coherent-phase averaging can be very large-up to 7-fold here, but variable. Our theory correctly predicts the SNR gain achieved from constructive averaging of *in vivo* human heart and leg spectra. The results are important for optimizing applications of 1H MRS for studying low-SNR metabolites in the torso (breast, heart) deleteriously affected by motion.

References

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