Dependence of the DTI signal on the presence of fiber crossing

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Introduction

The (sub)cellular structure of tissue, especially in the brain, can be probed using diffusion-weighted MR. Of the techniques employed, diffusion tensor imaging (*DTI)* is a powerful tool to delineate fiber directionality thus enabling to establish fiber connections and directions non-invasively, when there is a preferential orientation within the imaging voxel. In the case of "pure" fiber orientation, the typical model to characterize diffusion directionality is to use the diffusion tensor model, which is characterized by six degrees of freedom. When two or more divergent fiber orientations are present, the degrees of freedom are increased, but it is unclear if diffusion measurements have the necessary degrees of freedom to resolve the presence of multiple fibers. Several techniques have been developed to solve this problem by increasing the number of gradient directions [1],[2],[3]. In contrast, the aim of the present study was to examine if, based on the DT description, multiple fiber orientations present in a voxel will permit the measurement of more than 6 degrees of freedom.

Theory

It is assumed that the signal acquired for one fiber orientation in a voxel is given by the normalized diffusion signal $S^{(1)} / S_0^{(1)} = \exp\left(-\bar{u}^t \overline{\overline{D^{(1)}}} \bar{u} \cdot b\right)$ [4], where $\bar{u}$ the direction of the gradient, $\overline{\overline{D^{(1)}}}$ the matrix of diffusion of the fiber. If two fiber orientations are present, the signal acquired is the sum of two corresponding signals: $S_{sum}^{voxel} = \sum_{i \subset voxel} \alpha^{(i)} S_0^{(i)} \exp\left(-\bar{u}^t \overline{\overline{D^{(i)}}} \bar{u} \cdot b\right)$ (Eq. 1) where S^{voxel} is the signal acquired for the entire voxel, and $\alpha^{(i)}$ is the relative contribution of one fiber orientation ($\sum_{i \subset voxel} \alpha^{(i)} = 1$). The apparent diffusion signal acquired with a *DTI* experiment is $S_{acqu}^{voxel} = S_0^{(voxel)} \exp\left(-\bar{u}^t \overline{\overline{D^{(voxel)}}} \bar{u} \cdot b\right)$ (Eq. 2). By equating Eq. 1 and 2 it can be shown that the matrix of diffusion $\overline{\overline{D^{(voxel)}}}$ is a symmetric matrix with 6 degrees of freedom with an added dependence on the diffusion attenuation *b*, depending on relative fiber orientation: $D_{ij}^{(voxel)} = -\ln\left(\frac{S_0^{(1)} \alpha^{(1)}}{S_0^{(voxel)}} \exp\left[-D_{ij}^{(1)} \cdot b\right] + \frac{S_0^{(2)} \alpha^{(2)}}{S_0^{(voxel)}} \exp\left[-D_{ij}^{(2)} \cdot b\right] \right) \Big/ b = D_{ji}^{(voxel)}$

This approach can be generalized to more than two orientations, and it can be shown that the symmetry of the diffusion tensor matrix is preserved at a given *b*-value, hence the number of degrees of freedom remain constant at 6.

Methods

The effect of multiple fiber orientations of variable relative contribution on the diffusion signal was simulated using Matlab, assuming that the tensor description is valid for a single pure orientation [4]. The MR signal was simulated with 5% of noise along 6 directions at 5 different *b*-values from 0 to 3000 s/mm^2 .

The data are sorted by direction (u_i) and a plot of the signal as a function of the *b*-value is fitted with the following bi-exponential function: $S_{fit}(u_i, b) = \alpha \exp\left(-D_{u_i}^{(1)} b\right) + (1-\alpha)\exp\left(-D_{u_i}^{(2)} b\right)$. The parameter α was assumed to be the same for the six directions and the coordinate system was chosen to coincide with the eigenvectors of one of the fibers. The second fiber (without restriction in generality) was assumed to have the same diffusion characteristics, an angle of 90°, 45° and 22,5° in the plane XY and contribute 40% to the unattenuated signal. Twelve apparent diffusion coefficients (6 for each fiber) and the ratio of fibers are obtained by these six fits. In fact, the plots are first fitted with a mono-exponential function and afterwards with a bi-exponential function. The comparison of the χ^2 test of the two fits shows whether there are one or two fiber in the same voxel.

With this method it is possible to solve a problem of thirteen unknowns, which allows us to completely resolve the problem of two fibers crossing in a voxel.

Results and Discussion

The signal behaviour was well described by a bi-exponential function. The $\chi 2$ values were in general 2 to 8 times smaller than for the mono-exponential fit for an inter-fiber angle from 90° to 45° (Fig 1). For smaller angles the method was less robust ($\chi 2$ values were reduced by 1.5 to 3 times).

The comparison between the assumed *ADC* contour and the simulated one was in excellent agreement (Fig 2). It suggests that it is possible to determine if two fibers are present in the same voxel or not. The eigenvalues found corresponded well with the input for the main direction. There appears to be some mixing of the minor diffusion directions between the two fibers.

Without substantial increases in measurement time, it appears that fiber crossings can be detected by a direct measurement. Further improvements are possible by optimizing the number of *b*-values and their range.

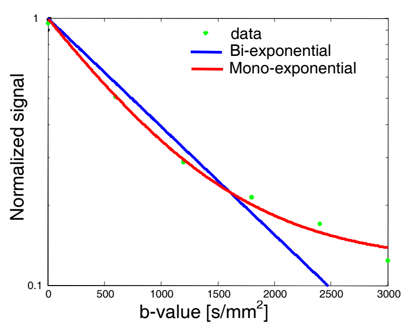

Figure 1: Fit for two fibers with an angle of 90° Plotted on a semi-log vertical scale

a)

b)

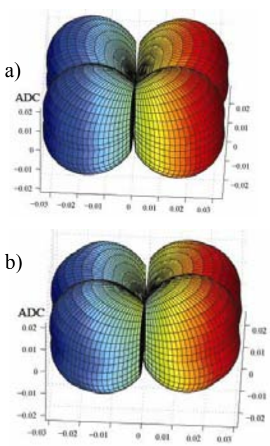

Figure 2: ADC contour of the two fibers with an angle of 90°. a) Assumed b) Simulated

Conclusion

We conclude that based on the diffusion tensor formalism for a given fiber orientations the presence of multiple fibers cannot be resolved at one diffusion attenuation only, but that additional information may be gathered by extending the measurement to multiple *b*-values, thus adding additional degrees of freedom to the measurement, which may be used to establish relative directionality for multiple orientations. The proposed approach obviates the need for acquisition at high angular resolution, instead emphasizes the need for multiple b values which in themselves bear important information on the physiological basis of the diffusion-weighted MR signal.

Acknowledgements

This study was supported by Centre d'Imagerie BioMédicale (CIBM) of the UNIL, UNIGE, HUG, CHUV, EPFL, the Leenaards and Jeantet Foundations and the Fonds National Suisse de la recherche scientifique (FNS)

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