Efficient Visualization of Fiber Tracking Uncertainty based on Complex Gaussian Noise

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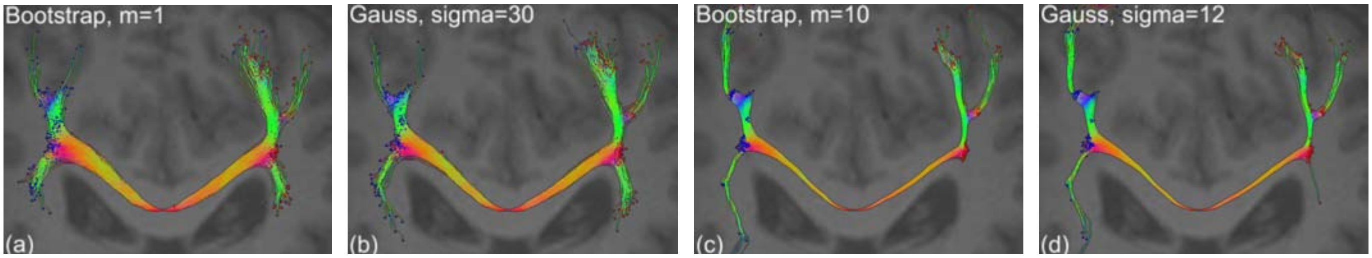

Fig. 1: *(a) Fibers derived from bootstrap method (sample size m=1). (b) Fibers generated with our new method with noise level chosen corresponding to noise in (a). (c) Bootstrap, m=10 (d) Gauss, noise corresponding to (c). For better orientation the axial slice intersecting most of the fibers of a T1-weighted data set is shown.*

Abstract: Visualizing the uncertainty of fiber tracking is an important new challenge in the area of Diffusion Tensor Imaging (DTI). In this paper, we present a new method that allows for an efficient computation of diffusion weighted images with user-defined noise. These are used to analyze the tracking uncertainty resulting from image noise. The main idea is to add complex Gaussian noise to the magnitude images. In contrast to the bootstrap method, our technique needs only a single data set so that the acquisition time and the time for computing the artificial data can be reduced dramatically. Our visualization of the resulting fiber sets as well as our measurements show that bootstrap noise can be simulated appropriately by complex Gaussian noise.

Introduction: For risk analysis prior to interventional treatment of brain tumors, it is important to identify the functional brain areas affected by the tumor and to estimate their connectivity. Fiber Tracking (FT) algorithms on Diffusion Tensor data have the potential to facilitate this task. However, the resulting fibers can differ from the true white matter pathways of the patient due to several factors. One determining factor, which is examined in this paper, is the susceptibility to noise in DTI measurements. Other errors, e.g., distortions, registration errors, image blur, resolution-based errors, or "incorrect" parameter settings in the FT algorithm also limit the precision of a FT connection path (*streamline*). Although an assessment of the FT with respect to its uncertainty is essential in clinical applications, only little work has been done in this area. In a recent paper by Jones et al. [1] a bootstrap method for visualizing the uncertainty of fiber orientation in conjunction with the trajectory data was presented. However, their method needs several repetitions of each image, resulting in a time-consuming acquisition step. To overcome this problem, we present a novel method that allows for generating Diffusion Weighted (DW) images with user-defined noise from a single data set.

Methods: As we compare our method with the bootstrap method [1,2], we will first give a quick recap of it.

Bootstrap method: Given a superset acquisition consisting of *n* repetitions of each DW image (with respect to a specific gradient direction), new DW images are created by drawing (with replacement) *m* images from each set consisting of *n* images, followed by averaging the *m* images. Note that *m*, the *sample size*, can be larger than *n*. Thus, a new data set can be determined in time $O(g \cdot m \cdot s) \in O(m \cdot s)$, where *s* denotes the number of slices in the data set and *g* the number of vectors of the gradient scheme (the big O notation is used to describe the asymptotic behaviour of a function). With this method the noise becomes lower with increasing *m*.

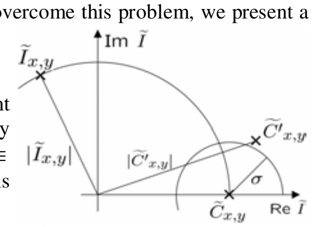

Fig. 2: *Complex Gaussian noise.*

Complex Gaussian Noise method: The noise distribution of acquired MR images, even after 2D inverse Fourier transformation is commonly assumed to be Gaussian [3]. However after magnitude calculation the data is no more complex and the noise is Rician distributed [4]. We now propose to add complex Gaussian noise to the magnitude images, so that the noise distribution is equivalent to those of standard MR images. Given a pixel value $|\widetilde{I}_{x,y}|$ of the magnitude image, we define a corresponding complex number $\widetilde{C}_{x,y} = |\widetilde{I}_{x,y}| + 0i$ because we can choose an arbitrary point on the circle of the magnitude (the magnitude determination of the complex MR signal $\widetilde{I}$ is rotation-invariant, see Fig. 2). For the moment, let us assume we would like to add noise with a complex Gaussian distribution of width σ to $\widetilde{C}_{x,y}$. The resulting complex number is $\widetilde{C'}_{x,y} = \widetilde{C}_{x,y} + \widetilde{N}(0,\Sigma)$ where $\widetilde{N}(0,\Sigma)$ denotes a normal distributed complex number with mean 0 and covariance matrix $\Sigma = \sigma^2 E$. Finally, the corresponding new magnitude value can be determined as $|\widetilde{C'}_{x,y}|$ (see Fig. 2). Now, let us assume the MR signal $\widetilde{I}$ is corrupted by Gaussian noise with variance $\sigma_1{}^2$. Further, let $\sigma_2{}^2$ denote the desired variance of the new image. It is well known that, if $X_1 \sim N(\mu_1, \Sigma_1)$ and $X_2 \sim N(\mu_2, \Sigma_2 - \Sigma_1)$ are independent variates, then $X_1 + X_2 \sim N(\mu_1 + \mu_2, \Sigma_2)$. As a consequence, σ^2 can simply be determined as $\sigma^2 = \sigma_2{}^2 - \sigma_1{}^2$, so that the standard deviation is $\sigma = \sqrt{\sigma_2{}^2 - \sigma_1{}^2}$. The complexity of our new approach is only in O(*s*), because all new images can be derived from the existing ones.

Evaluation: DW images were acquired in 30 gradient directions from a volunteer on a 3T Siemens head-scanner. Five equivalent data sets were acquired, which is only necessary for the bootstrap method. As source data S_1 to which we would like to add complex Gaussian noise, we take the average of the 5 registered data sets. We compare our technique with the bootstrap method by generating new DW images from S_1 that have the same noise as bootstrapped images. For that purpose, we determine the average noise σ_1 of S_1 [5]. Moreover, we compute a new data set S_2 by the bootstrap method. σ_2 for the Gaussian method is determined in accordance to the average of all 1000 bootstrapped data sets in order to yield a comparable noise level. Then, the difference noise σ can be added to images from S_1 as described above. For both data sets, fiber tracking is performed for a certain seed point *x* and the streamlines are stored. This process is repeated 1000 times for the same seed point *x*, so that we get two sets consisting of 1000 streamlines each.

Results: Fig. 1 gives a first impression that Bootstrap noise can be simulated appropriately by complex Gaussian noise. The streamline bundles in Fig. 1 (a) and (b) as well as in (c) and (d) are similar with respect to the paths, the variance and the branching points. The plot in Fig. 3 supports this observation. For all points lying on the streamlines that have the same geodesic distance *d* to the seed point *x*, their average distance to the superset trajectory (the streamline that results when we add no noise to S_1) is determined. As one can see, corresponding curves (Gauss sigma=30/Bootstrap m=1 and Gauss sigma=12/Bootstrap m=10) have a highly similar curve characteristic. Remaining small differences need to be investigated in future work. Note that for both methods the evaluation of fiber tracking uncertainty on low noise images is limited by the superset noise.

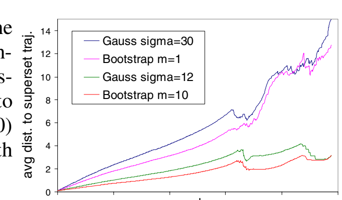

Fig. 3: *Average distance to superset trajectory.*

Conclusion: We have shown that our method constitutes an alternative to the Bootstrap method for analyzing tracking uncertainty. In contrast to the bootstrap method, it does not need multiple acquisitions of DTI data and hence makes the examination less strenuous for the patient.

References:

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