

# EFFECTS OF LOCAL SUSCEPTIBILITY GRADIENTS ON SPIRAL K-SPACE TRAJECTORY AND EFFECTIVE ECHO-TIME (TE<sub>eff</sub>)

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## INTRODUCTION:

Rapid imaging methods, such as echo-planar and spiral imaging often suffer severe distortions due to the presence of background field inhomogeneities. The constant component of the background inhomogeneity, particularly for high field MRI, leads to local blurring or geometric shifting, depending on the acquisition method. The gradient of the background magnetic field variation produces a local distortion, which leads to a shifting and skewing of the k-space trajectory. In this work, we analyze the background gradients in terms of their effect on the echo-time, TE, in gradient echo spiral imaging. We experimentally verify that the localized susceptibility gradients produce an effective local echo-time (TE<sub>eff</sub>).

## THEORY:

The presence of a local background field inhomogeneity,  $\Delta\omega(x)$ , modulates the signal,  $s(t)$ , received in gradient echo MRI as given in Eqn. 1. Existing methods such as

$$s(t) = \int m(x) e^{-i2\pi k(t) \cdot x} e^{-i\Delta\omega(x)(t+TE)} dx \quad (1), \quad \Delta\omega(x) = \Delta\omega(x_0) + \gamma G_B(x_0) \cdot (x - x_0) \quad (2)$$

conjugate phase reconstruction [2], assume that the inhomogeneity only affects the single point,  $x_0$ . If however, the magnetic field is not uniform around  $x_0$ , we must consider the linear component of the variation in the magnetic field. Taking a Taylor series expansion the background gradient,  $\Delta\omega(x)$ , can be expressed as Eqn 2. Considering the gradient ( $G_B$ ) term only, it can be shown that the distortion of the k-space trajectory induced by the background gradients at a point  $x_0$ , is  $k_{tot}$  (Eqn. 3). The first term in Eqn. 3 is the original k-space trajectory, while the next two terms have the effect of skewing and shifting the trajectory, respectively. The positive and

$$k_{tot}(x_0, t) = k(t) \pm \frac{\gamma}{2\pi} G_B \cdot t + \frac{\gamma}{2\pi} G_B \cdot TE \quad (3), \quad \frac{\gamma}{2\pi} |G_B| \cdot (TE \pm t') = C \cdot t'^{2/3} \quad (4)$$

negative sign on the skewing term represents the forward (out) and reverse (in) spiral cases, respectively. We define an “effective” echo-time (TE<sub>eff</sub>) as the time when the phase accrual due to the applied and background gradients is zero. For a slew rate limited gradient acquisition, this will occur when the radial location in k-space approximated as  $k_r = C \cdot t'^{2/3}$  equals  $k_B$ , as given in Eqn. 4. The effective echo time will be TE<sub>eff</sub> = TE  $\pm$  t', where t' is the solution to Eqn. 4. Figure 1 shows some theoretical examples of the shifting and skewing of the trajectory. The TE<sub>eff</sub> occurs when the shifted k-space trajectory crosses the k-space origin.

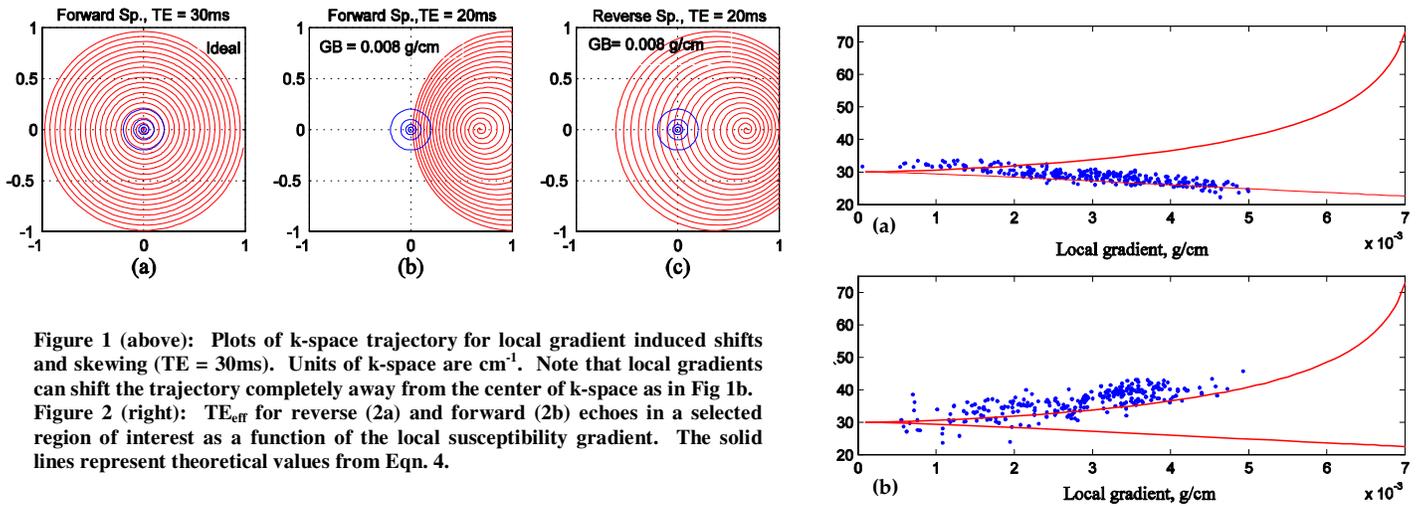


Figure 1 (above): Plots of k-space trajectory for local gradient induced shifts and skewing (TE = 30ms). Units of k-space are cm<sup>-1</sup>. Note that local gradients can shift the trajectory completely away from the center of k-space as in Fig 1b. Figure 2 (right): TE<sub>eff</sub> for reverse (2a) and forward (2b) echoes in a selected region of interest as a function of the local susceptibility gradient. The solid lines represent theoretical values from Eqn. 4.

## METHODS:

For two signal sources having different relaxivities,  $R2_a^*$  and  $R2_b^*$ , the percent signal difference (% $\Delta S$ ), for sufficiently small  $\Delta R2^* = R2_a^* - R2_b^*$ , can be approximated as: % $\Delta S = \Delta R2^* \cdot TE$ . By manipulating, in a phantom, the  $R2^*$  values, we can obtain a measure of the effective TE in the presence of the local magnetic field inhomogeneity. The phantom used was a large uniform container with an air-filled ball placed securely inside to provide a source of susceptibility. Two concentrations of CuSO<sub>4</sub> ( $R2^* = 0.53$ /mM/s) were used, with T2\* values of 65ms and 190ms measured using a multi-echo spin-warp GRE acquisition. The two solutions were each poured into the phantom and imaged successively using a spiral acquisition sequence that collected both forward and reverse echoes. The images were reconstructed using a spatially varying conjugate phase reconstruction [3]. In-plane gradients ( $G_B$ ) were computed by taking the derivative of the field maps along the x and y directions. TE<sub>eff</sub> was computed from the signal difference maps as  $TE_{eff} = \% \Delta S / \Delta R2^*$ .

## RESULTS:

Figure 2 is a plot of TE<sub>eff</sub> values as a function of in-plane susceptibility gradients in a selected region of interest in a slice chosen so that phase deviations due to susceptibility gradients did not require unwrapping, when calculating the field maps. The bold lines in Fig. 2 show the theoretical expected variation in effective echo time with varying local gradient as per Eqn. 4. It is clear that the experimental TE<sub>eff</sub> values are closely correlated with the predicted values. The TE<sub>eff</sub> values for the reverse echo (Fig 2a.) more closely track the theoretical curves than those for the forward echo (Fig 2b.). This is due to higher SNR achieved by reduced duration for evolution of background gradient inhomogeneity effects as well as less overall T2\* signal loss due to reduction, rather than lengthening, of the effective echo-time.

## DISCUSSION:

Distortions in the k-space trajectory due to local inhomogeneity have been previously analyzed, e.g. for the case of echo-planar imaging [4]. In this paper we present an experimental verification of the shifting of k-space trajectories and changes in TE due to local susceptibility gradients. The reverse spiral trajectory differs from the forward spiral acquisitions in that the phase accumulation associated with  $G_B$  evolves in reverse, skewing the k-trajectory back towards the “true” origin rather than away. This can be seen in Figure 1c where the trajectory now covers the origin of k-space. Due to the symmetry of the k-space trajectory, these analyses are independent of the direction of the background gradients.

## REFERENCES:

- [1] P. Bornert, B. Aldefeld, H. Eggers, Magn. Reson. Med. 44:479-484, 2000. [2] H. Schomberg, IEEE Trans. Med. Imaging, 18:481-495, 1999. [3] D.C. Noll, J.A. Fessler, IEEE Trans. Med. Imaging, 24(3): 325-336, 2005. [4] R. Deichmann, O. Josephs, C. Hutton, D.R. Corfield, R. Turner, Neuroimage, 15:120-135, 2002. Supported in part by NIH grants R01EB002603-01 and R01DA015410-01.