Registering Spherical Navigator Echoes using Spherical Harmonic Expansions to Measure 3D Rotations

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Introduction: The spherical navigator (SNAV) echo acquires complex *k*-space data on the surface of a sphere in order to measure 3D rigid-body motion in MRI. Analysis begins by registering the magnitude of two SNAVs to determine the 3D rotation between them. Several different methods to register SNAV data exist, each with specific capabilities and limitations. In this study we assessed the accuracy and reproducibility of measuring rotations in 3D using the spherical harmonic expansions of SNAV data. This technique was first introduced in ref. [1] using a simulated rotation in 1D, and showed promise as a non-iterative and potentially fast FFT-based algorithm. Our analysis employed the SOFT freeware package of C routines distributed under the GNU General Public License [2]. SOFT stands for "*SO(3)* Fourier transform", where *SO(3)* denotes the group of proper rotations of $\Re^3$ about the origin. Although computationally expensive, it is shown that registering SNAVs with this technique is accurate, precise, and robust.

Theory: The theory behind correlating SNAV data with spherical harmonics is outlined in [1] and follows from the work presented in [2]. In brief, two functions defined on the surface of a sphere, f_1 and f_2, can be approximated by a finite, linear series of spherical harmonics of degree l and order m (Y_l^m). The two functions represent the magnitude of two SNAVs acquired before and after a rigid body has undergone some rotation g, where $g \in SO(3)$. Under the rotation g, each Y_l^m is transformed into a linear combination of spherical harmonics with the same degree, l. As such, rotation of a function expressed by spherical harmonics is equal to multiplication by a semi-infinite block diagonal matrix, termed the Wigner *D*-function. Explicit expressions for these functions can be found in [2]. The pointwise multiplication of the spherical harmonic expansions of f_1 and f_2 corresponds to the *SO(3)* coefficients of f_1 correlated with f_2, $C(g)$. The discrete inverse SOFT of this multiplication yields the correlation $C(g)$ evaluated on a $2B \times 2B \times 2B$ grid, where B is the bandwidth of the inverse SOFT and the grid axes correspond to the three Euler angles. In theory, the coordinates of the maximum value on this grid correspond to the angles closest to the true 3D rotation $g \in SO(3)$ between f_1 and f_2.

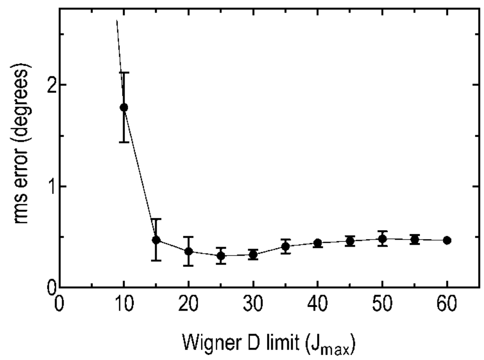

Fig. 1: Mean accuracy and precision in detecting the six rotations for increasing J_{max}.

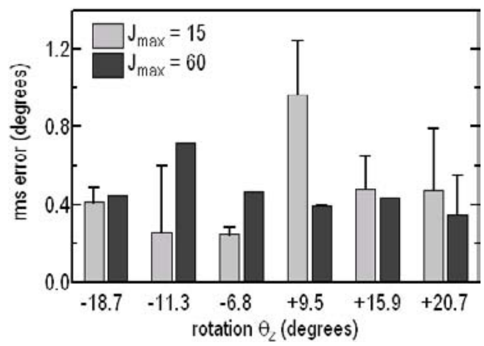

Fig. 2: Rotation measurements obtained with J_{max} values of 15 and 60; the error bars are the standard deviation across 32 repetitions.

Methods: SNAV data were acquired on a 1.5-T MRI scanner (GE Signa CV/I) using the variable-sampling density (VSD) SNAV trajectory described in [3] at a *k*-space radius of 1.6 cm^{-1} and 2058 points per hemisphere. Because this trajectory does not acquire points precisely on the colatitude-longitude grid required by the SOFT algorithm, the VSD SNAV data were interpolated prior to being correlated using the cssgrid (cubic spline sphere gridder) package from the ngmath library of NCL routines.

The VSD SNAVs were acquired of a human skull phantom filled with agar. The phantom was manually rotated about the SI axis to seven different positions (one baseline, six rotated), with rotations ranging from θ_z = -18.7° to +20.7°. At each position, a 3D image and 32 repetitions of an axial VSD SNAV were acquired. The rotated 3D images were retrospectively registered to the baseline image using a 3D rigid-body, Downhill-simplex-based algorithm, which provided a "true" measure of the rotation for calculating the error in the SOFT algorithm.

SNAV registrations were performed using the test_soft_fftw_correlate2 routine of the SOFT package, which takes as input the sample values of f_1 and f_2, the bandwidth of the two functions, the desired bandwidth of the inverse SOFT (B), and the maximum degree of Wigner-*D* coefficients to use (J_{max}), and outputs the maximum value on the 3D Euler-angle grid, that is, the measured 3D rotation. Registrations were performed using J_{max} values ranging from 5 to 60 in increments of 5 and a bandwidth of B = 256. Processing was performed by a SGI Altix 3700 equipped with a 1.5-GHz Itanium 2 CPU.

Results & Discussion: Figure 1 shows that as J_{max} is increased, both the mean accuracy and mean precision of the SOFT algorithm improve to a limit of approx. 0.50 +/- 0.04°. This limit is related to the resolution of the $2B \times 2B \times 2B$ Euler-angle grid: the average margin of error in the rotation that maximizes $C(g)$ is given by $\sqrt{3(\pi/2B)^2}$. Figure 2 shows the distribution of rotation measurements for J_{max} values of 15 and 60; note the high accuracy and precision in the J_{max} = 60 results. One of the exciting possibilities of registering SNAVs with spherical harmonic expansions is the potential to detect rotations at even higher accuracy by increasing B. However, increasing B also comes with the penalty of longer computation time and greater memory demands; for B = 256, each registration required 3-4 minutes to complete and 4.5 GB of RAM. Current developments are aimed at improving the efficiency and memory handling of the software. With further testing, this technique may become the registration method of choice for retrospective SNAV applications, such as inter-scan alignment of a patient's successive MRI examinations.

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References:

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