Comparison of Wavelets and a new DCT Algorithm for Sparsely Sampled Reconstruction

J. Plett[1], I. Lillo[1], M. Guarini[1], P. Irarrazaval[1]
[1]Department of Electrical Engineering, Pontificia Universidad Catolica de Chile, Santiago, Chile

INTRODUCTION

If an image can be represented in a sparse domain, it has been shown in previous work [1,2,3], that it is possible to reconstruct it from a small number of projections onto random subspaces. The reconstruction through a convex L_1 norm optimization can be accurate or even exact, depending on the sparsity or compressibility of the image. So far, domains used for the optimization algorithm have been based on the Wavelet transform [1,2,3,4]. We present a Discrete Cosine Transform (DCT) based approach and compare it to the Wavelet based strategies. The results show that the DCT based approach yields an increased PSNR index, lower time of computation and improved overall visible perception.

METHOD

Since MRI images are often sparse or compressible in some domain, reconstruction can be implemented as a convex L_1 norm minimization problem constrained to having a sparse or compressible representation and being consistent with the original samples. In general the problem can be stated as

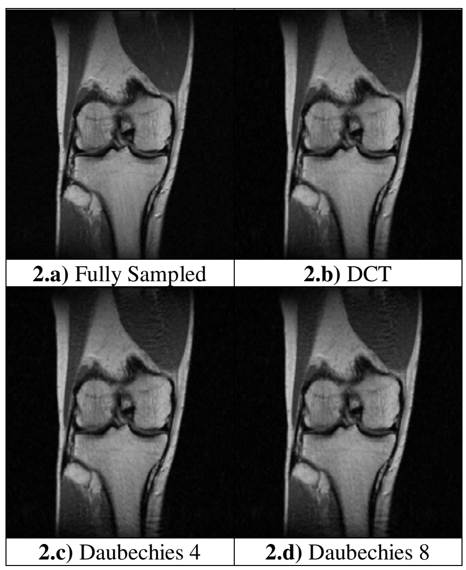
$$minimize \ \|\Psi(m)\|_1 \quad s.t \ F_\Omega m = y \qquad (1)$$

In equation 1 Ψ represents the transform, m the reconstructed image, F_Ω the sub sampled Fourier transform matrix and y the k-space samples. For the Wavelet variant of the minimization Ψ represents the Wavelet transform, whereas in the DCT variant it represents a windowed version of the DCT. For computing these minimization problems a variant of POCS was implemented.

RESULTS

In order to compare the DCT and Wavelet based approaches in MRI, two complex images of 256x256 pixels were obtained using a 0.5T Philips Gyroscan scanner. Two knee images were taken in the sagittal and coronal planes using 2DFT spin echo trajectories with TE=30ms and TR=500ms. The DCT approach was compared with Wavelet algorithms using Daubechies 4 and Daubechies 8 Wavelets. All three were implemented with an asymmetric 35% k-space sub sampling using 90 of 256 lines, with 53 central lines. The results were compared with a fully sampled reconstruction using the MSE and PSNR indices given in equations 2 and 3. The reconstructed images are shown in figures 1 and 2 while the performance indices for each method are shown in the table below.

1.a) Fully Sampled **1.b)** DCT **1.c)** Daubechies 4 **1.d)** Daubechies 8



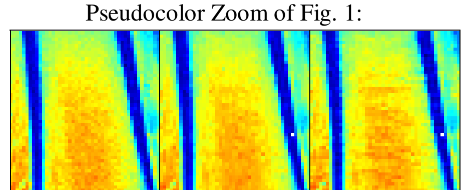
2.a) Fully Sampled **2.b)** DCT **2.c)** Daubechies 4 **2.d)** Daubechies 8

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}\left\|I_{orig}(i,j) - I_{rec}(i,j)\right\|^2 \qquad (2)$$

$$PSNR = 20\log_{10}\left(\frac{\Delta MAX_{I_{orig}}}{\sqrt{MSE}}\right) \qquad (3)$$

	DCT	Daub. 4	Daub. 8
MSE (%)	0.0338 0.0607	0.0463 0.0705	0.0461 0.0700
PSNR (dB)	34.71 32.17	33.35 31.52	33.36 31.55

Pseudocolor Zoom of Fig. 1:



3.a) Fully Sampled **3.b)** DCT **3.c)** Daub. 4

CONCLUSION

The proposed DCT based method outperforms standard Wavelet based algorithms in PSNR, time and overall visible perception. PSNR rates are clearly better for the DCT method for a variety of MRI images. The proposed method yields an average increase of more than 1dB over Wavelets and, thus, substantially surpasses the MPEG committee's informal 0.5dB threshold for perceptible improvement. Regarding computational times it can be stated that at less than 1.5mins per image the DCT based algorithm is inherently at least three times faster than Wavelet variants due to far more efficient gradient calculations. The DCT method shows an excellent reconstruction, visibly reducing artefacts in the phase encoding direction and enhancing sharpness compared to Wavelet based methods, while maintaining the general advantage of reducing the number of necessary samples of this reconstruction method. In conclusion we state that the DCT can be a superior alternative to Wavelets for this kind of image reconstruction and should be considered in future work.

REFERENCES

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