# Optimal Data Sampling for Noquist ciné imaging

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## Introduction

Faster acquisition of ciné imaging sequences remains of critical importance for continued development of advanced applications in cardiac MRI. Parallel imaging and partial-Fourier techniques offer important benefits and are now widely available, but have limitations in attainable acceleration, and may not always be compatible with phase-sensitive data such as phase velocity encoded studies. Further improvement in acquisition speed, compatible with flow encoding, may be obtained through reduced field of view methods, which use the presence of static field of view (FOV) regions in dynamic images as prior information for data reduction. We have recently proposed the "Noquist" rFOV method [1], which uniquely preserves full spatiotemporal resolution by a direct inversion approach and is flexible with respect to dynamic FOV size and location.

In a dynamic reconstruction problem with $N_P$ time points and a phase encoding FOV with $N_S$ static and $N_D$ dynamic points, Noquist only requires a total of $N_{NQ}=N_S+N_P \cdot N_D$ total phase encodings (compared to $N_{CNV}=N_P \cdot (N_S+N_D)$ for conventional ciné imaging) to reconstruct the entire ciné sequence acquiring static region(s) once and the dynamic region for each time point. This acceleration is similar to that offered by other rFOV methods like UNFOLD [2], but unlike other rFOV methods the subset selection of phase encoding views used is flexible and may be optimized for SNR, similar to data subset selection in parallel imaging.

In this abstract we report on initial results of our ongoing research on optimization of the Noquist method. The experiments were designed to obtain phase encoding selection patterns with optimal SNR for additive white noise, testing a hypothesis that patterns which are (1) spaced evenly across (*k*,t)-space and (2) ensure complete sampling of a cartesian *k*-space grid, i.e., with all phase encoding views acquired at least once during the cardiac cycle, will correspond with a well-conditioned inversion problem.

## Materials and Methods

To determine optimal data sampling for Noquist ciné imaging, we performed optimization by exhaustive search for minimal-size reconstruction problems, with $N_P=N_S=N_D=2, 3, 4$. Three separate optimization criteria were considered and compared in this process: reconstruction matrix condition number and mean and maximum noise amplification in the dynamic region, compared to conventional full-grid acquisition. The exhaustive search visited all (16, 3,375, 9,834,496) possibilities to select $N_{NQ}$ (6, 12, 20) acquisition views out of $N_{CNV}$ (8, 18, 32) for $N_P$ (2, 3, 4) phases, under the constraint for ciné imaging that equal numbers of views are selected at each time point of the sequence. The direct-inversion reconstruction matrix was computed if the selection was invertible, and SNR performance was evaluated for each case by noise propagation calculation following the method described in [1], assuming additive, independent, identically distributed white noise for all acquired data. Practical sampling pattern algorithms, generalized from the method proposed in [1] were compared with the optimal patterns for SNR performance. Results were extrapolated for several well-conditioned algorithmic data selection patterns to confirm stable characteristics of the pattern algorithms and to obtain noise propagation curves for practical image dimensions, where exhaustive search optimization is computationally intractable.

## Results

Optimization of the minimal-size ciné sequences by exhaustive search revealed SNR optimality of the evenly spaced pattern of which an example is in Figure 1 as well as optimality of symmetric permutations, supporting the hypothesis stated above. All three investigated optimization criteria lead to the same results. Figure 2 shows an example image from a 16-frame Noquist reconstruction, with SNR factor for the dynamic region of 1.6956. Figure 3 shows an upper bound of the attainable SNR as a function of the number of phases.

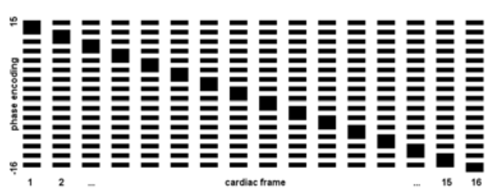


**Figure 1:** k-Space sampling grid (black=acquired view, white=omitted view) for 32 phase-encodings and 16 cardiac phases and a 50% dynamic FOV.

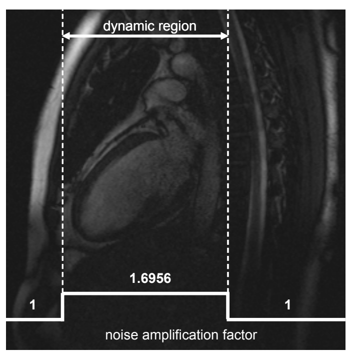


**Figure 2:** Frame of a Noquist reconstruction for a ciné acquisition with 256 phase-encodings and 16 cardiac phases and a 50% dynamic FOV.

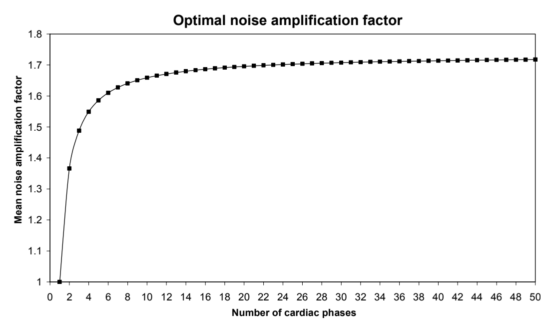


**Figure 3:** Optimal mean noise amplification factor as a function of the number of cardiac phases (from 1 to 50).

## Discussion and Conclusions

Optimization by exhaustive search for a minimal-size reconstruction problem has been performed extrapolating conclusions to practical image dimensions for which exhaustive search optimization is computationally intractable. All optimization criteria converged to same optimal patterns.

The optimization study shows SNR penalties comparable to half-Fourier imaging or parallel imaging techniques. The study confirms that stable rFOV reconstructions using NoQuist may be obtained using algorithmic data selection as outlined in [1] with marginally suboptimal SNR for a variety of different data set sizes. This observation suggests that optimization of data selection may not be a necessary step in all situations, and flexible ad-hoc algorithms offer a practical solution in many cases.

## References

1. Brummer ME, Moratal-Pérez D, et al. Noquist: Reduced Field-of-View Imaging by Direct Fourier Inversion. MRM 2004; 51:331–342.

2. Madore B, Glover GH, et al. Unaliasing by Fourier-Encoding the Overlaps Using the Temporal Dimension (UNFOLD), Applied to Cardiac Imaging and fMRI. MRM 1999; 42:813–828.