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## Does The Principle Of Reciprocity Hold At High Field MR?

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The principle of reciprocity is one of the basic and most important theorems in electromagnetic theory, and has far-reaching implication in optics, magnetic resonance imaging as well as spectroscopy. As formulated in antenna theory, the reciprocity theorem states that the transmit field is identical to receive sensitivity [1]. Hoult and Richards [2] prescribes that the magnetic field produced at a point in space by a unit current in a radiofrequency (RF) coil is proportional to the electro-motive-force induced in the coil by a magnetic dipole at the same point in space. Thus, receive sensitivity can be calibrated using measured local  $B_1$  field introduced by the receive coil, and is widely used in the quantification of <sup>1</sup>H MR spectra [3] and parallel imaging reconstruction [4]. Additionally, the signal-to-noise ratio is one of the most important MR parameter which directly affects the accuracy and precision of MRI and MRS quantification for research and clinical applications. The reciprocity theorem provides a unique method for estimating signal-to-noise ratio for MRI and MRS. However, at high field MR, several papers have reported contradictory results about the relationship between transmit field and receive sensitivity, as well as reciprocity theorem. It is very crucial to clarify these problems for high field MRI and MRS.

The transmit field is given by one of the circularly polarized components of  $B_1$  (typically  $B_1^+$ ), whilst the reception sensitivity is given by the other component ( $B_1^-$ ). Generally, the transmit field is identical to the receive sensitivity at low field MR. Recently, computer simulations prove that transmit field are not equal to receive sensitivity at high field MR [4-6]. Recently, Wang et al. show that receive sensitivity is significant different from transmit field for phantom and in vivo brain at 3 T [7]. Both computer simulation and experimental results demonstrate that the receive sensitivity is distinguished from transmit field at high field MR. What does the difference between transmit field and receive sensitivity means? What reasons (high frequency, wave behavior and correlated time-space) induce the differences? Whether the receive sensitivity can be estimated by the transmit field in accord with their new relationship.

The reciprocity principle in NMR is validated for arbitrary probe geometry [8]. Insko et al also evidence the effectiveness of reciprocity principle for near, intermediate, and radiation zone field of a magnetic dipole [9]. Both of them confirm that the transmit field is identical to the reception sensitivity. Recently, based on an experiment of small current loops, Hoult illustrates that in accord with reciprocity principle transmit field and receive sensitivity should be depicted by  $B_1^+$  and  $B_1^-$ , respectively, at higher static field strengths [10]. Ibrahim gives an analytical evaluation for Hoult's conclusion at 8 T [4]. These contradictory reports on the relationship between transmit field and reception sensitivity, claim the same conclusion that the reciprocity principle is held at high field MR. But Tofts indicates that the significant difference means reciprocity failure at high field MR [11]. It is not clear whether this reciprocity is held at the high field MR or not? Completely opposite views are used to explain the significant difference between transmit field and receive sensitivity at high field MR.

What is truth about reciprocity principle at high field MR? If the principle is valid, then why are the results from refs. [4-6] not consistent with refs. [8,9], in regard to the relationship between transmit field and receive sensitivity? In accord with Maxwell equations and reciprocity principle, the derived relationship between the transmit field and receive sensitivity should be consistent. If not, what would be the significance of such field-dependent validity of the principle? What is responsible for the failure of reciprocity principle at high field strength? We propose to employ classical electromagnetic theory, and phantom MRI experiments to clarify these questions.

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